COURSE CODE DCE 058
MATHEMATICS II

Instructions

Answer question one and any other four from section two.

Question one (30mks)

a) If f(x) = $\frac{3}{4-x}$ and g(x) = $\frac{1}{x}$ find

i. (fog) (x)
ii. (gof) (x) (4mks)

b) Find the derivatives of the following from first principles.

i. Cos x
ii. Cos x + sin x
iii. Log x
iv. 2ln x (8mks)

c) Integrate the following expressions with respect to x

i. $\frac{1}{2x}$

ii. e^{4-5x}

iii. $\frac{3x}{(x-1)(x-2)}$

iv. $\frac{1}{3x+1}$ (8mks)

d)

i. Given that $\log_a^{(m)} = r$ and $\log_a^{1/n} = s$ show that $m=n\,a^{r+s}$ (3mks)

ii. Express $\frac{2}{(x+1)(x2-x-1)}$ into partial fractions (4mks)

iii. If $\log_{10}^{2} = a$ show that $\log_8^5 = \frac{(1-a)}{3a}$ (3mks)

Question two (10mks)

If $\int_1^a 3(x+1)^2\,dx = a^3 + 11$, find the possible value of a.

Question three (10mks)

i. Differentiate from first principles

$f(x) = x^3 - 2x$ (6mks)

ii. Cos x + sin x (2mks)

iii. $2\sin\theta - 3\cos\theta$ (2mks)

Question four (10mks)

Use Simpson's rule with five ordinates to find an approximate value for

$$\int_o^{\pi} \sqrt{sin\theta}\,d\theta$$

Question five (10mks)

The area enclosed by the curve $y = 4x - x^2$ and the line y=3 is rotated about the line y=3. Find the volume of the solid generated.

Question six (10mks)

i. A spherical balloon is blown up so that its volume increases at constant rate of $2cm^3/s$. find the rate of increase of the radius when the volume of the balloon is 50 cm^3. (4mks)

ii. A vessel containing water is in the form of an inverted hollow cone with a semi-vertical angle of 30^o. there is a small hole at the vertex of the core and the water is running out at a rate of $3cm^3/s$. find the rate at which the surface area is contact with the water is changing when there are 81π cm^3 of water remaining in the cone. (6mks)