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(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER MAIN EXAMINATIONS

FOR THE DEGREE
OF
BACHELOR OF ENGINEERING ELECTRICAL ENGINEERING

COURSE CODE: **SPH 414**

COURSE TITLE: **QUANTUM MECHANICS II**

DATE: THURSDAY 28TH APRIL, 2022 **TIME:** 12:00 PM - 2:00 PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 Hours

Answer question ONE and any TWO of the remaining.
Symbols used bear the usual meaning.

MMUST observes ZERO tolerance to examination cheating

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 NARKS)

(a) (i) State the Pauli's exclusion principle (2 marks)

(ii) Determine the possible values of the quantum numbers n, l and m_l for an electron in a hydrogen atom if $\hat{L}^2 Y_l^m(\theta,\phi) = 12\hbar^2 Y_l^m(\theta,\phi)$. (3 marks)

(b) Prove that the matrix element of the Pauli matrices anti-commute and they follow the commutation relation for a harmonic oscillator, i.e. (5 marks)

$[\sigma_x, \sigma_y] = 2i\sigma_z$, $[\sigma_y, \sigma_z] = 2i\sigma_x$ $[\sigma_z, \sigma_x] = 2i\sigma_y$.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(c) Obtain the first order shift in energy of a perturbed system whose time independent Hamiltonian $H = H^{(0)} + \lambda H'$, where $H^{(0)}$ is the unperturbed Hamiltonian, H' is the perturbation and λ characterizes the strength of the perturbation. (5 marks)

(d) Show that the first-order effect of a time dependent perturbation varying sinusoidal in time, leads to emission and absorption of radiation. (5 marks)

(e) Use the classical definition of the angular momentum to derive the expressions for the components $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ in the Cartesian coordinate. (5 marks)

(f) The electron neutron scattering from a heavy nucleon can be represented by a potential

$$V(r) = \begin{cases} -V_0 & r < R \\ 0 & r > R \end{cases} \qquad V_0 > 0$$

Calculate Differential cross section in the lowest order of V (5 marks)

QUESTION TWO (20 MARKS)

(a) Consider the following relations;

$$\mathbf{L}_+ = \mathbf{L}_x + i\mathbf{L}_y \qquad \mathbf{L}_- = \mathbf{L}_x - i\mathbf{L}_y$$

$$\mathbf{L}_+ |l,m\rangle = \hbar\sqrt{l(l+1) - m(m+1)}\,|l,m+1\rangle$$

$$\mathbf{L}_- |l,m\rangle = \hbar\sqrt{l(l+1) - m(m-1)}\,|l,m-1\rangle$$

Standard basis $|1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$; $|0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$; $|-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

i. Compute the matrix representation of the angular momentum component L_x and L_y for a system with orbital quantum number $l = 1$ in the basis of eigenvector of L_z. (13 marks)

ii. Prove the following relation for the angular momentum operator

$\left[L^2, L_+\right] = 0$ (7 marks)

QUESTION THREE (20 MARKS)

(a) (i) Describe the basis of the variational method (4 marks)

(ii) The Hamiltonian for harmonic oscillator is given by

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \frac{1}{2}m\omega^2 x^2$$

Use variation method to obtain the ground state energy of the oscillator assuming a trial function $\psi(x) = e^{-\alpha x^2}$ where α is a variation parameter. (10 marks)

Take $\int_{-\infty}^{\infty} \exp\left(-2\alpha x^2\right) dx = \left(\frac{2\pi}{4\alpha}\right)^{1/2}$; $\int_{-\infty}^{\infty} x^2 \exp\left(-2\alpha x^2\right) dx = \frac{1}{8}\left(\frac{2\pi}{\alpha^3}\right)^{1/2}$

(b) Use the WKB approximation to obtain the energy levels of a linear harmonic oscillator (6 marks)

QUESTION FOUR (20 MARKS)

(a) The differential cross section in the first Born approximation is

$$\frac{d\sigma}{d\Omega} = \left|f(\theta,\phi)\right|^2 = \frac{-\mu^2}{4\pi^2\hbar^4}\left|\int e^{i\vec{q}\cdot\vec{r}} V(\vec{r}\,')\, d^3r'\right|^2$$

where $\vec{q} = \vec{k}_0 - \vec{k} = 2k\sin\left(\theta/2\right)$ and $\hbar \vec{k}_0$ and $\hbar \vec{k}$ are the linear momenta of the incident and scattered.

i. Calculate the elastic scattering amplitude $f(\theta,\phi)$ for a spherically symmetric potential $V(\vec{r}\,')$ with $\vec{q} = \vec{k}_0 - \vec{k}$ directed along the z-axis. (9 marks)

ii. Calculate the differential cross section in the first Born approximation for a Coulomb potential

$$V(r) = \frac{Z_1 Z_2 e^2}{r}$$

where $Z_1 e$ and $Z_2 e$ are the charges of the projectile and target particles, respectively. (6 marks)

Take $\int_0^{\infty} \sin(qr)\, dr = \lim_{\lambda \to 0} \int_0^{\infty} e^{-\lambda r} \sin(qr)\, dr = \frac{1}{q}$

(b) The spin-orbit coupling $\mathbf{L.S}$ expressed in terms of the total angular momentum **J**, the orbital angular momentum **L** and the spin angular momentum **S** is

$$\mathbf{L.S} = \frac{1}{2}\left(|\mathbf{J}|^2 - |\mathbf{L}|^2 - |\mathbf{S}|^2\right)$$

Calculate the possible values of $\mathbf{L.S}$ for L=3 and S=1/2 (5 marks)

QUESTION FIVE (20 MARKS)

(a) Using the first order time-dependent perturbation theory, calculate the probability of finding the system at the time t in the state $|1\rangle$ and the probability of finding it in sate $|2\rangle$

(8 marks)

(b) Calculate the first and second-orders corrections to the energy eigenvalues of a linear harmonic oscillator with the square term λx^2 added to the potential. Discuss the condition for the validity of the approximation. (12 marks)

Position operator $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a}_+ + \hat{a}_-)$