



**MASINDE MULIRO UNIVERSITY OF  
SCIENCE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS**

**2021/2022 ACADEMIC YEAR**

**FIRST YEAR FIRST SEMESTER MAIN EXAMINATIONS**

**FOR THE DEGREE**

**IN**

**BACHELOR OF SCIENCE (SST, SMT, SME, ETS, BIO, SCB, BTE, SAR,  
SEV, DPE)**

**COURSE CODE: BCS 121**

**COURSE TITLE: DISCRETE STRUCTURES II**

**DATE: TUESDAY 26<sup>TH</sup> APRIL, 2022      TIME: 8:00-10:00AM**

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**INSTRUCTIONS TO CANDIDATES**

- Answer Question **ONE** and any other **TWO** questions.
- Do all the rough work in the answer booklet

**TIME: 2 hours**

### QUESTION ONE (30 MARKS)

a) A class has 12 boys and 4 girls. Suppose three students are selected at random from the class.

Find the probability  $p$  that they are all boys.

(4 Marks)

b) What is a predicate? Give two examples of predicates.

(2 Marks)

c) Let  $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$  be Boolean matrices. Find the Boolean

matrices:

i)  $A \wedge B$

ii)  $A \vee B$

(4 Marks)

d) Suppose that the probability sample space of a given neuron being fired in a neural network system and recording the number of the neuron is  $A = \{1, 2, 3, 4, 5, 6\}$ . If the elementary for getting each of the numbers have been established as  $P(1) = 1/12, P(2) = 1/12, P(3) = 1/3, P(4) = 1/6, P(5) = 1/4, P(6) = 1/12$ , compute  $P(E)$  if the event  $E$  is the number of the neuron being an even number.

(6 marks)

e) i) Find the first four terms of the sequences defined by the recurrence relations  $a_n = 6a_{n-1}, a_0 = 2$

(2 Marks)

iii) What is the solution of the recurrence relation  $a_n = 8a_{n-1} - 16a_{n-2}$  with  $a_0 = 1$  and  $a_1 = 6$ ?

(6 Marks)

f) Given that  $P(X)$  is the statement 'X spends more than four hours revising for discrete structures II every weekend" where the universe of discourse for  $x$  consists of all first year computer students. Express each of the qualifications below in English

(i)  $\exists X P(X)$

(1 Mark)

(ii)  $\forall X P(X)$

(1 Mark)

(iii)  $\exists X \neg P(X)$

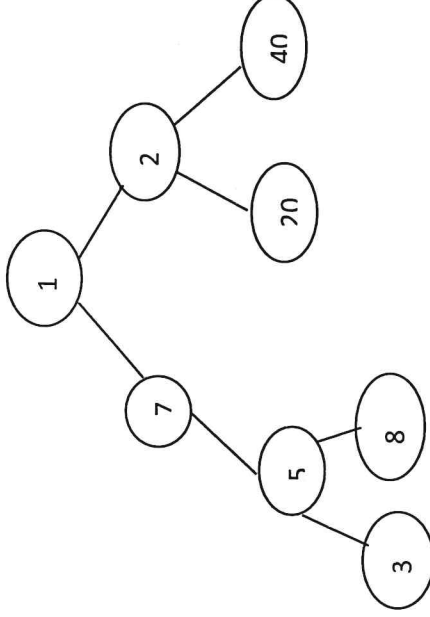
(1 Mark)

(iv)  $\forall X \neg P(X)$

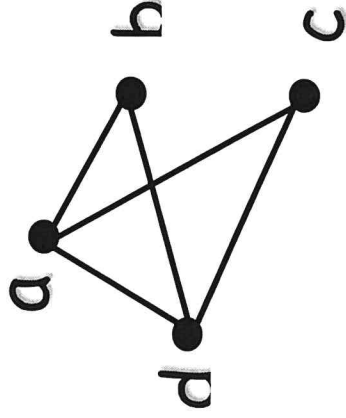
(1 Mark)

### QUESTION TWO (20 MARKS)

a) Form a binary search tree for the data 16, 24, 7, 5, 8, 20, 40 and 3 in the given order  
(4 Marks)



- b) Consider an undirected simple graph that has vertices  $a, b, c, d,$  and  $e$  of degree  $4,3,3,2,2.$   
 i) How many edges does it have? (3 Marks)
- ii) Draw this graph. (5 Marks)
- c) What is the adjacency matrix  $A_G$  for the following graph  $G$  based on the order of vertices  $a, b, c, d$ ? (4 Marks)



- d) What is the largest possible number of vertices in a data storage path graph with 70 edges and all vertices of degree at least 3 (4 Marks)

### QUESTION THREE (20 MARKS)

- a) The Kenya Bureau of standards discovered that there were fake microchips being sold in the market and hence decided to be testing all microchips before accepting them. The test machines have not been calibrated for some time hence at times give faulty results. The result is recorded as positive if the test machine gives it as genuine. From the sample to be tested the probability of picking a genuine micro chip is  $0.9$  and it testing negative is  $0.05$ . The probability of a fake microchip test positive from the sample is  $0.2$ .
- i) Illustrate the probability of the test results by using a tree diagram (3 marks)
- ii) Find the probability that the test results is positive (3 marks)
- iii) Find the probability of the test being correct (3 marks)
- iv) what is the probability that microchip testing positive is genuine (5 marks)
- b) A problem in discrete structures II is given to three students whose chance of solving it are  $\frac{1}{2}, \frac{1}{3},$  and  $\frac{1}{4}$  respectively. What is the probability that at least one of them solves it correctly (6 marks)

#### QUESTION FOUR (20 MARKS)

- a) Let  $P(x) = x + 2 = 5$ . Write down the following sentences and their meaning
- i)  $\exists xP(x)$  (4 Marks)
  - ii)  $\forall xP(x)$  (4 Marks)
- b) Consider the second-order homogeneous recurrence relation  $a_n = a_{n-1} + 2a_{n-1}$  with initial conditions  $a_0 = 2, a_1 = 7$ .
- i) Find the next three terms of the sequence. (3Marks)
  - ii) Find the general solution. (5 Marks)
  - iii) Find the unique solution with the given initial conditions. (4 Marks)
- c) Define the following terms
- i) Binary tree (4 Marks)
  - ii) Planar graph

#### QUESTION FIVE (20 MARKS)

- a) State the two cases in complexity theory. (2 marks)
- b) Translate into English the predicate logic used in artificial language  $\exists x(P(x) \wedge \neg Q(x))$ , where the  $P(x)$  says that  $x$  is a bird and  $Q(x)$  says that  $x$  can fly. Take the universe of discourse to be all living creatures. Suggest a witness to the existential quantifier (that is, a creature that makes the sentence true). (2 marks)
- c) Explain the Depth- First search Algorithm. (7 Marks)
- d) Find the number of edges and Hamiltonian circuits of a complete graph with 15 edges. (4 Marks)
- e) For what positive integers  $n$  is a complete graph with  $n$  vertices both eulerian and Hamiltonian? (5 Marks)