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(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
TOWN CAMPUS - SCHOOLBASED

SECOND YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE

BACHELOR OF EDUCATION (SCIENCE AND ARTS)

COURSE CODE: **STA 241**

COURSE TITLE: **PROBABILITY AND DISTRIBUTION MODELS**

DATE: 22ND APRIL 2022 **TIME: 9.00-11.00AM**

A. INSTRUCTIONS TO CANDIDATES

- Answer question **ONE** and **ANY OTHER TWO** questions.

Time 2 Hours

MMUST observes ZERO tolerance to examination cheating

QUESTION ONE (30Marks)

a) Define the following terms:

i. Random variable

ii. Sample space

iii. Probability density function

iv. Cumulative distribution function. (4marks)

b) The probability density function of random variable X is given by

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & ;-1 < x < 1 \\ 0 \quad ; & otherwise \end{cases}$$

Find

i. $E(X)$ (3marks)

ii. $Var(X)$ (4marks)

iii. $Var(5X+10)$ (2marks)

c) A random variable Y has probability density function given by

$$f(y) = \begin{cases} p^y(1-p)^{1-y} & ; y = 0,1 \\ 0 & ; otherwise \end{cases}.$$

i. Obtain the moment generating function (m.g.f) (3marks)

ii. Use the m.g.f obtained in (i) to find mean and variance of Y. (4marks)

d) In a given supermarket, 60% of the customers pay by credit card. Find the probability that in a randomly selected sample of 10 customers

i. Exactly 3 pay by credit card. (2marks)

ii. At least eight pay by credit card. (3mks)

e) The moment generating function of a random variable X from normal population is e^{4t+6t^2}

Find $p(X > 8)$ (5mks)

QUESTION TWO (20marks).

a) Consider the probability density function given by,

$$f(x) = \begin{cases} k(8x+2); & 1 \le x \le 5 \\ 0 & elsewhere \end{cases}$$

Where k is a constant. Calculate the:

i. Value of k. (2 marks)

ii. $E(x)$ (1 mark)

iii. $Var(x)$ (3 marks)

iv. C.D.F, $F(x)$ (2 marks)

v. $P(-0.5 < x < 2)$ (2 marks)

b) The probability density function of a random variable X is given by

$$f(x) = \begin{cases} (1-p)^{x-1}p \,; x = 1,2,3,\ldots \\ 0 \qquad ; \ otherwise \end{cases}$$

i. Show that the factorial m.g.f. is $M_X(t) = \dfrac{pt}{1-qt}$ (3marks)

ii. Use the m.g.f above to show that $E(X) = \dfrac{1}{p}$ and $Var(X) = \dfrac{q}{p^2}$ (7marks)

QUESTION 3 (20Marks)

a) Find the m.g.f for the distribution whose probability mass function is given by;

$$\text{p}(x) = \begin{cases} \dfrac{1}{6}\left(\dfrac{5}{6}\right)^x ; \ x = 0,1,2,3,\ldots \\ 0 \ ; \ otherwise \end{cases}$$

Use the m.g.f technique to determine the mean and variance of the distribution. (8mks)

b) In a region, the number of people who become ill from a given disease is a random variable having Poisson distribution $f(x) = \dfrac{e^{-2}2^x}{x!}; x = 0,1,2$

Find the probability of

i. Two illnesses in a year. (3marks)

ii. At most four illnesses in 4 years. (4marks)

c) The moment generating function of a random variable X is $e^{4(e^t-1)}$. Find $p(\mu - 2\sigma < x < \mu + 2\sigma)$ (5marks)

QUESTION 4 (20marks)

a) Juma is playing a board game in which he needs to throw a six with an ordinary die in order to start the game. Find the probability that

i. Write the required p.d.f (1mk)

ii. Four attempts are needed to obtain a six (3mks)

iii. At least two attempts are needed. (3mks)

iv. He is successful in throwing a six in five or fewer attempts. (3mks)

b) A random variable X is normally distributed with mean $\mu = 50$ and variance $\sigma^2 = 100$.

i. $p(X < 65)$ (3mks)

ii. $p(X > 72)$ (3mks)

iii. $p(33 < X < 45)$ (4marks)

QUESTION 5 (20marks)

a) The random variable X has probability density function given by $f(x) = \begin{cases} \lambda e^{-\lambda x}; x > 0 \\ 0; otherwise \end{cases}$

i. Show that the m.g.f. is given by $M_X(t) = \dfrac{\lambda}{\lambda - t}$ (4marks)

ii. Using the M.g.f above obtain the mean and variance of X. (6marks)

b) A random If e^{3t+8t^2} is the moment generating function of a random variable X.

Find

i. The mean and variance of X (3marks)

ii. $E(X^2 + 2X)$ (3marks)

iii. $P(-1 < X < 9)$ (4marks)