2600

(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATION

2021/2022 ACADEMIC YEAR

(MAIN EXAMINATIONS)

FIRST YEAR SECOND SEMESTER EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ENGINEERING
(MIE, ECE, CSE & SRT)

COURSE CODE: MAT 103

COURSE TITLE: PURE MATHEMATICS II

DATE: 29^{th} April, 2020 TIME: 3:00 PM - 5:00 PM

INSTRUCTIONS TO CANDIDATES:

- *Answer Question ONE (COMPULSORY) and ANY OTHER TWO questions.*
- *Do not write on the question paper.*

Time: 2 Hours

MMUST observes ZERO tolerance to examination cheating

This paper consists of 3 printed pages. Please turn over.

QUESTION ONE (COMPULSORY) [30 MARKS]

(a) Given two matrices **A** and **B**, simplify $(A-B)^2 + 3B(2B-A) + A^2$ **[3 marks]**

(b) Evaluate **[3 marks]**

$$\frac{3+6i}{2+3i}$$

(c) Prove $\frac{d}{dx}(sinhx) = coshx$ **[3 marks]**

(d) Find the rank of the matrix below **[3 marks]**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

(e) A particle moves in a straight line so that after t seconds its acceleration is given by $a(t) = 4t^2 + 2t + 6$. If its position at time $t = 0$ is $5m$ and the velocity is $10m/s$;

(i) Determine velocity at any time t. **[3 marks]**

(ii) Find the displacement at any time t. **[3 marks]**

(f) Evaluate $(4-i)^3 + (3+2i)$ **[3 marks]**

(g) Find the value of x for which **A** is a singular matrix **[4 marks]**

$$A = \begin{bmatrix} x^2 & x+6 \\ 1 & 1 \end{bmatrix}$$

(h) Evaluate by partial fractions $\int \frac{5x+7}{(x+1)(x+2)}dx$ **[5 marks]**

QUESTION TWO [20 MARKS]

(a) Solve the following $e^{2+\frac{\pi}{2}i}$ using Euler's formula **[3 marks]**

(b) Show that $AB \neq BA$. If **[4 marks]**

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

(c) Find x, y if $(1-3i)^2 - 5(x+iy) = x + iy$ **[4 marks]**

(d) Given the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$ Find

(i) The determinant of A. **[2 marks]**

(ii) Matrices of minors and Co-factors of A. **[4 marks]**

(iii) Adjoint of A and inverse of A. **[3 marks]**

QUESTION THREE **[20 MARKS]**

(a) Differentiate the functions

(i) $y = \frac{1-cosx}{sinx}$ **[4 marks]**

(ii) $y = e^{x^2}cos3x + sin^2x$ **[4 marks]**

(b) Solve the following system of equations using Cramer's rule **[7 marks]**

$$\begin{aligned} 5x - 6y + 4z &= 15 \\ 7x + 4y - 3z &= 19 \\ 2x + y + 6z &= 46 \end{aligned}$$

(c) Find $(1+i)^6$ using De Moivre's theorem **[5 marks]**

QUESTION FOUR **[20 MARKS]**

(a) Convert the following into polar form $\sqrt{3} - i$ **[4 marks]**

(b) Differentiate $f(x) = (2x+1)^2(x^3 - x)^3$ **[4 marks]**

(c) Find the determinant of $A = \begin{bmatrix} 2 & -3 & 4 \\ 5 & 1 & -6 \\ -7 & 8 & -9 \end{bmatrix}$ using the second row. **[4 marks]**

(d) Given that $y = x^4 + 3x^3$, find $\frac{d^3y}{dx^3}$ **[3 marks]**

(e) Evaluate $\int_0^{\frac{\pi}{2}} sin^2xcos^3xdx$ **[5 marks]**

QUESTION FIVE **[20 MARKS]**

(a) Evaluate $\int_0^{\pi} sin^2x$ **[3 marks]**

(b) Find $\frac{dy}{dx}$ given $y = ln[(x^2+1)(5x+9)]$ **[3 marks]**

(c) Show that if $x \neq 0$ then $y = \frac{1}{x}$ satisfies the equation **[4 marks]**

$$x^3y'' + x^2y' = xy$$

(d) Find the six roots of $z = 8$ and graph these roots in the complex plane. **[6 marks]**

(e) Differentiate $y = x^{3x+2}$ using Napierian logarithms **[4 marks]**