750

(The University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER MAIN EXAMINATIONS

FOR THE DEGREE

IN

BACHELOR OF SCIENCE (SST, SMT, SME)

COURSE CODE: **MAT 202**

COURSE TITLE: **LINEAR ALGEBRA II**

DATE: 29/04/2022 **TIME: 3.00PM – 5.00PM**

INSTRUCTIONS TO CANDIDATES

- Section A is compulsory any other THREE questions from section B
- Do all the rough work in the answer booklet

TIME: 2 hours

QUESTION ONE (30 MARKS)

a) Diagonalize the matrix $\begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$. (5 Marks)

b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ by use of the Cayley-Hamilton theorem.

(4 Marks)

c) Given that $B = \{(1,2,1),\ (2,9,0),\ (3,3,4)\}$ is a basis for an inner product space $V \subseteq \mathbb{R}^3$. Find the coordinate matrix of $w = (5,-1,9)$ with respect to B. (5 Marks)

d) Show that if $S = \{u_1, u_2, \dots, u_n\}$ is an orthornomal set of vectors in an inner product space V, then for every $v \in V$, the vector$w = v - < v, u_1 > u_1 - < v, u_2 > u_2 - \cdots - < v, u_n > u_n$ is orthogonal to each u_i. (3 Marks)

e) Verify that the $S = \left\{(0,1,0),\ \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right),\ \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)\right\}$ is an orthornormal set I $\mathbb{R}^3$.

(3 Marks)

f) Find the value of a scalar a so that the polynomial $P(x) = -1 + ax$ is a unit vector in P_1 with the integral inner product on the interval $[0,1]$. (4 Marks)

g) i) Show that if u is orthogonal to v, then every scalar multiple of u is orthogonal to v.

(2 Marks)

ii) Find a unit vector orthogonal to both $v_1 = (1,1,2)$ and $v_2 = (0,1,3)$ in $\mathbb{R}^3$ (4 Marks)

QUESTION TWO (20 MARKS)

a) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$. $B = \{(1,3,\ (-2,4))\}$ and $B' = \{(1,1,1), (2,2,0), (3,0,0)\}$ are bases of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. Further $T(x_1, x_2) = (x_1 + 2x_2, -x_1, 0)$. Find the matrix of T with respect to B and B'. (8 Marks)

b) Find a matrix P that orthogonally diagonalizes the matrix $A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. (7 Marks)

c) Identify the curve whis represented by the following quadratic equation by first putting it into standard conic form. (5 Marks)

$$x^2 + 2xy + y^2 - x + y = 0$$

QUESTION THREE (20 MARKS)

a) For the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 3 & 2 \\ 3 & 0 & 1 \end{bmatrix}$

i) Write down the characteristic polynomial

ii) Write down the characteristic equation

iii) Find the eigen values and eigen vectors corresponding to each eigen value.

iv) Find the basis for each eigen space. (12 marks)

b) If V is an n −dimensional vector space and $[.]_B$ is a coordinate mapping with respect to basis B, Show that

i) $[u + v]_B = [u]_B + [v]_B$ for $u, v \in V$

ii) $[\lambda v]_B = \lambda[v]_B$ for $\lambda \in \mathbb{R}, v \in V$ (4 Marks)

c) Verify that the following is an inner product in P_2

$$\langle p(x), q(x) \rangle = a_0 b_0 + a_1 b_1 + a_2 b_2$$

where $p(x) = a_0 + a_1 x + a_2 x^2$ and $q(x) = b_0 + b_1 x + b_2 x^2$. (4 marks)

QUESTION FOUR (20 MARKS)

a) Show that a square matrix A_{nxn} is diagonalizable if and only if A has n linearly independent eigenvectors. (8 Marks)

b) If $v_1, v_2, \ldots, v_n$ are eigenvectors associated with distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of a matrix A_{nxn}, show that the set $\{v_1, v_2, \ldots, v_n\}$ is linearly independent. (6 Marks)

c) Let V be the vector space of polynomials over $\mathbb{R}$ and define $\langle f(t), g(t) \rangle = \int_0^1 f(t) g(t) dt$.

Find the angle θ between u and v if $u = 2t - 1$ and $v = t^2$. (6 Marks)

QUESTION FIVE (20 MARKS)

a) Consider the following bases of $\mathbb{R}^2$; $B = \{(1,0), (0,1)\}$ and $B' = \{(1,2), (2,3)\}$. Let $T(x_1, x_2) = (x_1 + 7x_2, 3x_1 - 4x_2)$. Find

i) Find A, the matrix of representation of T with respect to B.

ii) Find A' i.e. matrix of representation of T with respect to B' by using $A' = P^{-1}AP$ where P is transition matrix from B' to B. (7 Marks)

b) State and prove the Cauchy – Schwarz inequality. (7 Marks)

c) Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$. Find all eigenvalues of A and the corresponding eigenvectors. Also find an invertible matrix P such that $P^{-1}AP = D$ (where D is diagonal matrix). (6 Marks)