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(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

MAIN EXAMINATIONS

SECOND SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: MAT 864
COURSE TITLE: FLUID MECHANICS I

TIME: 3 HOURS DATE: 28TH APRIL 2022, 9:00AM - 12:00AM

Instruction to the candidates:
Answer question ONE (COMPULSORY) and any other TWO questions
Time: 3 hours

This paper consists of 4 printed pages. Please turn over.

SECTION I: Answer ALL the questions in this section

QUESTION ONE - 30 MARKS (COMPULSORY)

(a) Differentiate between isobaric and adiabatic processes. **[4 mks]**

(b) Show, that the differential equation of a perfect gas is

$$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0$$

[4 mks]

(c) Show, that the sonic velocity for an isothermal process is given by $C = \sqrt{RT}$ while that of an adiabatic process is given by $C = \sqrt{\gamma RT}$. **[5 mks]**

(d) Show, that the area-velocity relationship for a compressible flow is given by the Hugoniot relation

$$\frac{dA}{A} = \frac{dV}{V}(M^2 - 1) \qquad (\because M = \frac{V}{c}).$$

[5 mks]

(e) A large vessel, fitted with a nozzle, contains air at a pressure of 2500 kN/m^2 (abs.) and at a temperature of 20°C. If the pressure at the outlet of a nozzle is 1700 kN/m^2 find the velocity of air flowing at the outlet of the nozzle. Take: $R = 287$ J/kg K, and $\gamma = 1.4$. **[5 mks]**

(f) In a duct in which air is flowing, a normal shock wave occurs at Mach number of 1.5. The static pressure and temperature upstream of the shock wave are 170 kN/m^2 and 23°C respectively. Determine pressure, temperature and Mach number downstream of the shock, and the strength of the shock. **[5 mks]**

(g) Show, that the mass rate of flow through a venturimeter is given by

$$m = \rho_2 A_2 V_2 = \rho_2 A_2 \sqrt{\frac{\left(\frac{2\gamma}{\gamma - 1}\right)\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}{1 - \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}\left(\frac{A_2}{A_1}\right)^2}}$$

[4 mks]

(h) A 25 mm diameter venturimeter is fixed in a 75 mm diameter pipe to measure the rate of flow of a gas. If the absolute pressure at the inlet and the throat of venturimeter are equivalent to 1010 mm and 910 mm of mercury, determine the volumetric flow rate of gas. Assume the flow to be isentropic, $\gamma = 1.4$ and $\rho_1 = 1.6$ kg/m^3. **[5 mks]**

(i) Using the Euler's equation of compressible fluid, $\frac{dp}{\rho} + VdV + gdz = 0$, show that the area velocity relationship for a compressible flow is given by

$$\frac{dA}{A} = \frac{dV}{V}(M^2 - 1)$$

, where M is the Mach number. **[4 mks]**

SECTION II: Answer any TWO questions from this section

QUESTION TWO - 15 MARKS

(a) A 120 mm diameter pipe reduces to 60 mm diameter through a sudden contraction. When it carries air at 25°C under isothermal condition, the absolute pressures observed in the two pipes just before and after the contraction are 480 kN/m^2 and 384 kN/m^2 respectively. Determine:

(i) Densities of the two sections, **[3 mks]**

(ii) Velocities of the two sections, and **[5 mk]**

(iii) Mass rate of flow through the pipe **[2 mk]**

Take $R = 287$ J/kg K.

(b) In the case of air flow in a conduit transition, the pressure, velocity and temperature at the upstream section are 35 kN/m^2, 30 m/s and 150° C respectively. If at the downstream section the velocity is 150 m/s, determine the pressure and the temperature if the process followed isentropic. Take $\gamma = 1.4$, $R = 290$ J/kg K. **[5 mk]**

QUESTION THREE - 15 MARKS

(a) In case of isentropic flow of a compressible fluid through a variable duct, show that

$$\frac{dV}{V} = \frac{1}{\left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]} \frac{dM}{M}$$

where γ is the ratio of specific heats and M is the Mach number. Use the continuity equation $\rho AV =$ constant, energy equation $c_p T + \frac{V^2}{2} =$ constant. **[9 mks]**

(b) A supersonic nozzle is to be designed for air flow with Mach number 3 at the exit section which is 200 mm in diameter. The pressure and temperature of air at the nozzle exit are to be 7.85 kN/m^2 and 200 K respectively. Determine the reservoir pressure and temperature and the throat area. Take $\gamma = 1.4$. **[6 mks]**

QUESTION FOUR - 15 MARKS

(a) Consider a body immersed in a compressible fluid. Assume that the flow around the body is frictionless, that is, $\left(\frac{\gamma}{\gamma - 1}\right) \frac{p}{\rho g} + \frac{V^2}{2g} + z =$ constant. Show, that the stagnation pressure is given by

$$p_s = p_0 \left[1 + \frac{\gamma - 1}{2} M_0^2\right]^{\frac{\gamma}{\gamma - 1}},$$

where M_0 is the Mach number at a point in the fluid at the same level with the tip of the body, s **[9 mks]**

(b) Show, that the compressibility correction factor in (a) above is given by

$$\left[1 + \frac{M_o^2}{4} + \frac{2 - \gamma}{24} M_o^2 + \ldots\right]$$

[5 mks]

QUESTION FIVE - 15 MARKS

(a) Show, that the mass rate of flow through a horizontal venturimeter is given by

$$m = \rho_2 A_2 \sqrt{\frac{\left(\frac{2\gamma}{\gamma - 1}\right)\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}{1 - \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} \times \left(\frac{A_2}{A_1}\right)^2}}$$

[10 mks]

(b) A 25 mm diameter venturimeter is fixed in a 75 mm diameter pipe to measure the rate of flow of a gas. If the absolute pressure at the inlet and the throat of venturimeter are equivalent to 1010 mm and 910 mm of mercury, determine the volumetric flow rate of gas. Assume the flow to be isentropic, $\gamma = 1.4$ and $\rho_1 = 1.6$ kg/m^3. **[5 mks]**