(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (IN MATHEMATICS)

COURSE CODE: MAT 222
COURSE TITLE: ADVANCED CALCULUS

DATE: APRIL 25, 2022 TIME: 3.00 PM - 5.00 PM

Instruction to the candidates:

Answer question ONE (COMPULSORY) and any other TWO questions
Time: 2 hours

This paper consists of 3 printed pages. Please turn over.

SECTION A: Answer ALL the questions in this section

QUESTION ONE - 30 MARKS (COMPULSORY)

(a) Define continuity of the function $f(x,y)$ at the point (a,b). [2 mks]

(b) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}.$$ [3 mks]

(c) The possible error involved in measuring each dimension of a rectangular box is $\pm 0.01cm$. The dimensions of the box are $x = 50cm$, $y = 20cm$ and $z = 15cm$. Use differentials to estimate the propagated error in the calculated volume of the box. [4 mks]

(d) Given $Z = x^2y^3$ where $x = Sint$ and $y = Cos3t$ find $\frac{dz}{dt}$. [4 mks]

(e) Find the directional derivative of $f(x,y) = 3x^2 - 2y^2$ at the point $(-\frac{3}{4}, 0)$ in the direction of the vector $v = \frac{3}{4}i + j$. [4 mks]

(f) Evaluate the iterated integral

$$\int_1^2 \int_0^4 (x^2 - 2y^2 + 1)dxdy$$

[4 mks]

(g) Find the equation of the tangent plane and the normal to the surface $Z = x^2 + 5xy - 2y^2$ at the point $(1,2,3)$ [5 mks]

(h) Determine if the sequence $\{\frac{2n}{1+n}\}$ is monotonic. [4 mks]

SECTION B: Answer any TWO questions from this section

QUESTION TWO - 20 MARKS

(a) A ball is dropped from a height of 9 feet and begins bouncing up two-thirds of the previous distance on each bounce. Find the total vertical distance traveled by the ball. [5 mks]

(b) Use the integral test to check for convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{2}{3n+5}$$

[5 mks]

(c) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} 3(x-2)^n$$

[5 mks]

(d) Find the Taylor series polynomial of degree 4 for the function $f(x) = \frac{1}{x}$ about the point $x = 1$ **[5 mks]**

QUESTION THREE - 20 MARKS

(a) Given $f(x, y) = xe^{x^2 y}$ find f_x and f_y at the point $(1, ln2)$. **[4 mks]**

(b) The temperature at any point (x, y) in a steel plate is $T = 500 - 0.06x^2 - 1.5y^2$. In what direction from the point $(2, 3)$ does the temperature increase most rapidly? What is this rate of increase? **[5 mks]**

(c) Suppose that x and y are related by the equation $F(x, y) = 0$, where it is assumed that $y = f(x)$ is a differentiable function of x. Derive an expression for $\frac{dy}{dx}$ and hence find $\frac{dy}{dx}$ given $y^3 + y^2 - 5y - x^2 + 4 = 0$. **[6 mks]**

(d) Determine and distinguish the extrema of $f(x, y) = x^2 + 6xy + 10y^2 - 4y + 4$ **[5 mks]**

QUESTION FOUR - 20 MARKS

(a) The profit obtained by producing x units of product A and y units of product B is approximated by the model $P(x, y) = 8x + 10y - (0.001)(x^2 + xy + y^2) - 10000$. Find the production level that maximizes profit and determine the maximum profit. **[7 mks]**

(b) Using Lagrange multipliers find the minimum value of the function $f(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y + 4z = 49$ **[7 mks]**

(c) Find the limit of the sequence $\{n^3 e^{-n}\}$ **[3 mks]**

(d) Show that $z = e^{-y} sinx$ satisfies the Laplace equation $z_{xx} + z_{yy} = 0$. **[3 mks]**

QUESTION FIVE - 20 MARKS

(a) Evaluate

$$\int_R \int sin\theta dA$$

where R is the first quadrant region lying inside the circle given by $r = 4cos\theta$ and outside the circle given by $r = 2$ **[7 mks]**

(b) Evaluate

$$\int_0^4 \int_0^{\pi} \int_0^{1-x} xsinydzdydx$$

[6 mks]

(c) Given the functions $u^2 + xv^2 = x + y$ and $v^2 + yu^2 = x - y$ where u and v are functions of the independent variables x and y, find $\frac{\partial u}{\partial x}$ using Jacobians. **[4 mks]**

(d) Find the total differential of $z = 2x^2 y + xy^3$ **[3 mks]**