12

(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

MAIN CAMPUS

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER EXAMINATIONS
SPECIAL/SUPPLEMENTARY

FOR THE DEGREE OF
BACHELOR OF ELECTRICAL ENGINEERING (ECE)

COURSE CODE: **MAT 401**

COURSE TITLE: **COMPLEX ANALYSIS I**

DATE: **Thursday, 6th October, 2022** **TIME:** **3-5 PM.**

INSTRUCTIONS TO CANDIDATES

- Answer question ONE (COMPULSORY) and any other TWO questions

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Find all the values of $(-8i)^{\frac{1}{3}}$ (4 Marks)

b) Find all complex numbers z such that $\sec Z = 2i$. (4 Marks)

c) Given that $f(z) = u + iv$ is an analytic function and suppose

$v(x, y) = 2xy - \dfrac{y}{x^2 + y^2}$ Use the Milne-Thompson method to find $f(z)$ and hence $u(x, y)$. (5 Marks)

d) Let C be the circle $|z| = 4$ traversed once in the counterclockwise direction. Evaluate

$$\int_C \frac{\cos z}{z^2 - 6z + 5} dz$$

(4 Marks)

e) Let f be defined by $f(z) = \dfrac{1}{z(z-1)}$ and f is analytic in the region $0 < |z| < 1$. Find the Laurent series for f valid in this region. (4 Marks)

f) Find the residues of $f(z) = \dfrac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all of its poles in the finite plane

(5 Marks)

g) Prove that the sufficient conditions for $f(z) = u(x, y) + iv(x, y)$ to be analytic in the region R (4 Marks)

QUESTION TWO (20 Marks)

a) State and prove Lioville's theorem (4Marks)

b) Find two Laurent series expansions for $f(z) = \dfrac{1}{z^3 - z^4}$ that involves powers of z.

Use the regions $0 < |z| < 1$ and $|z| > 1$ (6 Marks)

c) Consider the transformation $T : Z \to (1+i)z + 3 - 4i$ defined for any Z.

(i) Find the image $A'B'C'D'$ under T of the square $ABCD$ with vertices $A = 1 + i$, $B = -1 + i$, $C = -1 - i$ and $D = 1 - i$ (5 Marks)

(ii) How do the areas of $A'B'C'D'$ and $ABCD$ compare? (3 Marks)

(iii) Find the fixed points of the transformation T (2 Marks)

QUESTION THREE (20 Marks)

a) Evaluate $\displaystyle\int_0^\infty \frac{dx}{x^4 + 1}$ (6 Marks)

b) Show that z^5+6z^3-10 has exactly two zeros counting multiplicities in the annulus $2<|z|<3$ (4 Marks)

c) Determine the following limits (5 Marks)

$$\lim_{z\to 1+i}\left(\frac{z^4+2iz^2+8}{z^2-3iz-3+i}\right)$$

d) Evaluate $\int_C \frac{z+1}{z^3-4z}dz$ where C is the $|z-2|=\frac{3}{2}$ (5 Marks)

QUESTION FOUR (20 Marks)

a) Determine if the function $U(x,y)=e^x(y\cos 2y+x\sin 2y)$ is harmonic (4 Marks)

b) Suppose v is harmonic to conjugated to u and u is harmonic conjugated to v. Show that u and v must be constant functions. (4 marks)

c) Show that $f(z)=|z|^2$ is differentiable at the point $z_0=0$ but not at any other point (3 Marks)

d) Suppose $f(z)$ is analytic in the multiply connected region □ , lying inside the simple closed curve C and outside the non-intersecting simple close curve C_1 and C_2 lying entirely in C show that $\int_C f(z)dz=\int_{C_1} f(z)dz+\int_{C_2} f(z)dz$ (5 Marks)

e) Let $z=1-i$. Find z^{10} (4 Marks)

QUESTION FIVE (20 Marks)

a) Let C be the curve $y=\frac{1}{x^2}$ from the point $z=1+i$ to the point $z=3+\frac{i}{9}$. Find $\int_C z^2 dz$

(3 Marks)

b) Evaluate the integral $\int_C \frac{e^z}{z^3}dz$ where C is any positively oriented closed curve around origin. (3 Marks)

c) Find the integral $\int_C \frac{\cos Z}{e^Z-1}dz$ where C is the rectangle with sides $x=\pm 1, y=-\pi$ and $y=3\pi$ (5 Marks)

d) Find a bilinear transformation that maps points $z=0,-i,-1$ into $w=i,1,0$ respectively. (4 Marks)

e) Find the Laurent's series expansion for the function $f(z)=\frac{1}{z^2-3z+2}$ in the region $|z|<1$ (5 Marks)