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(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

(Main Campus)

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

EXAMINATION

THIRD YEAR FIRST SEMESTER SPECIAL/SUPLIMENTARY EXAMINATIONS

FOR THE DEGREE OF

BACHELOR SCIENCE IN: GEOSPATIAL INFORMATION SCIENCE

COURSE CODE: DPG 300

COURSE TITLE: NUMERICAL METHODS

DATE: 28/7/2022 **TIME: 8 – 10AM**

Instructions to Candidates

- This paper contains **FIVE** (5) questions
- Answer ALL questions in Section A and **ANY TWO** in Section B

MMUST observes ZERO tolerance to examination cheating

This Paper Consists of 3 Printed Pages. Please Turn Over →

SECTION A: Answer ALL questions [*30 Marks*]

Question ONE

a) 1 Divide a polynomial $f(x)=x^4-5x^3+5x^2+5x-6$ by the monomial factor $x-2$. Is $x=2$ a root? *[3 Marks]*

b) Given the data

0.90 1.42 1.30 1.55 1.63 1.32 1.35 1.47 1.95 1.66 1.96 1.47 1.92 1.35 1.05 1.85 1.74 1.65 1.78 1.71 2.29 1.82 2.06 2.14 1.27

Determine

i. the mean, *[3 Marks]*
ii. the standard deviation, *[3 Marks]*
iii. the variance, *[1 Mark]*
iv. the coefficient of variation, *[2 Marks]*
v. the 95% confidence interval for the mean. *[3 Marks]*
vi. construct a histogram using a range from 0.6 to 2.4 with intervals of 0.2. *[5 Marks]*

c) Use least-squares regression to fit a straight line to

x	0	2	4	6	9	11	12	15	17	19
y	5	6	7	6	9	8	7	10	12	12

Along with the slope and intercept, compute the standard error of the estimate and the correlation coefficient. *[10 Marks]*

SECTION B: Answer any TWO questions [*40 Marks*]

Question TWO

Given the system of equations

$$-3x_2+7x_3=2$$
$$x_1+2x_2-x_3=3$$
$$5x_1-2x_2=2$$

(a) Compute the determinant. *[4 Marks]*
(b) Use Cramer's rule to solve for the x's. *[6 Marks]*
(c) Use Gauss elimination with partial pivoting to solve for the x's. *[8 Marks]*
(d) Substitute your results back into the original equations to check your solution [*2Marks*]

Question THREE

Employ inverse interpolation to determine the value of x that corresponds to $f(x)=0.85$ for the following tabulated data:

x	0	1	2	3	4	5
y	0	0.5	0.8	0.9	0.941176	0.961538

Note that the values in the table were generated with the function

$$f(x) = x^2/(1+x^2)$$

i). Determine the correct value analytically. ***[1 Marks]***
ii). Use cubic interpolation of x versus y. ***[5 Marks]***
iii). Use inverse interpolation with quadratic interpolation and the quadratic formula. ***[6 Marks]***
iv). Use inverse interpolation with cubic interpolation and bisection. ***[6 Marks]***
v). For parts (ii) through (iv) compute the true percent relative error. ***[2 Marks]***

Question FOUR

Linear algebraic equations can arise in the solution of differential equations. For example, the following differential equation derives from a heat balance for a long, thin rod (Fig. 4-1).

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0 \qquad (4\text{-}1)$$

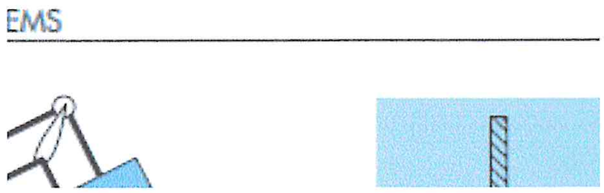

Figure 4-1. A noninsulated uniform rod positioned between two walls of constant but different temperature. The finite difference representation employs four interior node.

where T = temperature (^0C), x = distance along the rod (m), h' = a heat transfer coefficient between the rod and the ambient air (m^{-2}), and T_a = the temperature of the surrounding air (^0C). This equation can be transformed into a set of linear algebraic equations by using a finite divided difference approximation for the second derivative,

$$\frac{d^2T}{dx^2} = \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2} \qquad (4\text{-}2)$$

where T_i designates the temperature at node *i*. This approximation can be substituted into Eq. (4-1) to give

$$T_{i+1} - 2T_i + T_{i-1} + h'\Delta x^2(T_a - T) = 0 \qquad (4\text{-}3)$$

This equation can be written for each of the interior nodes of the rod resulting in a tridiagonal system of equations. The first and last nodes at the rods ends are fixed by boundary conditions.

a) Develop an analytical solution for Eq. (4-1) for a 10-m rod with T_a = 20, T(x = 0) = 40, T(x = 10) = 200 and h' = 0.02.

b) Develop a numerical solution for the same parameter values employed in (a) using a finite-difference solution with four interior nodes as shown in Fig. 4-1 (x = 2 m)