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MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR, FIRST SEMESTER

FIRST YEAR MAIN EXAMINATIONS

FOR THE DEGREE OF

MASTER OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: MAT 862
COURSE TITLE: NUMERICAL ANALYSIS I

DATE: 18^{TH} *APRIL* 2023 TIME:

Instruction to the candidates:

Answer ANY THREE questions
Use six decimal places unless stated otherwise
Time: 3 hours

This paper consists of 4 printed pages. Please turn over.

QUESTION ONE - 20 MARKS

(a) Given the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

(i) Find all the eigenvalues and the corresponding eigenvectors. **[5 mks]**

(ii) Verify that $S^{-1}AS$ is a diagonal matrix, where S is the matrix of eigenvectors. **[3 mks]**

(b) Perform four iterations of the Newton Raphson method to find an approximate value of $17^{\frac{1}{3}}$. Take the initial approximation as $x_0 = 2$. **[5 mks]**

(c) Solve the system of equations below using Gauss Seidel method given $x^0 = 0$. Perform three iterations **[7 mks]**

$$\begin{aligned} 2x - y &= 7 \\ -x + 2y - z &= 1 \\ -y + 2z &= 1 \end{aligned}$$

QUESTION TWO - 20 MARKS

(a) Derive the Newton Raphson method for solving nonlinear equations. **[4 mks]**

(b) Evaluate

$$\int_0^1 \frac{1}{1+x}\,dx$$

by Simpson's rule with 4 equal subdivisions **[5 mks]**

(c) Perform five iterations of the bisection method to obtain the root of the equation $x^3-5x+1 = 0$ in the interval [0,1]. **[4 mks]**

(d) Use Doolittle decomposition method to solve the system below **[7 mks]**

$$\begin{aligned} 2x + y + z - 2v &= -10 \\ 4x + 2z + v &= 8 \\ 3x + 2y + 2z &= 7 \\ x + 3y + 2z - v &= -5 \end{aligned}$$

QUESTION THREE - 20 MARKS

(a) Using the data $sin(0.1) = 0.09983$ and $sin(0.2) = 0.19867$, find an approximate value of $sin(0.15)$ by Lagrange interpolation. Obtain the truncation error. **[6 mks]**

(b) Use the power method to find the dorminant eigenvalue and the corresponding eigenvector of the matrix below given the starting vector as $\mathbf{X} = [1, 1, 1]^T$. Perform 4 iterations. **[7 mks]**

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

(c) Perform two iterations of the Newton Raphson method to solve the system of nonlinear equations below.

$$x^2 + xy + y^2 = 7$$
$$x^3 + y^3 = 9$$

Let

$$x_0 = 1.5$$
$$y_0 = 0.5$$

[7 mks]

QUESTION FOUR - 20 MARKS

(a) State the intermediate value theorem. **[2 mks]**

(b) Obtain the least squares polynomial approximation of degree one and two for the function $f(x) = x^{\frac{1}{2}}$ in $[0, 1]$ **[6 mks]**

(c) Obtain the Taylor series approximation about $x = 1$ for the function $f(x) = \frac{1}{1+x^2}$. Find the bound on the error if this approximation is to be used in $[1, 1.4]$ **[5 mks]**

(d) Fit the quadratic splines to the following data with $M(0) = f''(0) = 0$. Hence find an estimate of $f(2.5)$ **[7 mks]**

x	0	1	2	3
F(x)	1	2	33	244

QUESTION FIVE - 20 MARKS

(a) Perform the Crout's decomposition of the matrix below into a product of two triangular matrices $\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}$ **[7 mks]**

(b) Solve the system below by Cholesky's method **[7 mks]**

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 8x_2 + 22x_3 = 6$$
$$3x_1 + 22x_2 + 82x_3 = -10$$

(c) Construct the forward difference table for the data below. Hence, find the interpolating polynomial and an approximation to the value of $f(7)$ **[6 mks]**

x	0.5	1.5	3.0	5.0	6.5	8.0
y	1.625	5.875	31.0	131.0	282.125	521.0