10

(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2022/2023 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: **MAT 869**

COURSE TITLE: **COMPLEX ANALYSIS I**

DATE: **18TH APRIL 2023** **TIME:** **2.00-5.00 PM**

INSTRUCTIONS TO CANDIDATES

- Attempt ANY THREE questions

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE: (20 MARKS)

a) Find the residues of $f(z)=\dfrac{z^2+2z}{(z+1)^2(z^2+4)}$ at all its poles and hence evaluate

$\oint_c f(z)dz$ **(10 Marks)**

b) Using examples distinguish between a pole and a zero of a complex function **(5 Marks)**

c) Find the maximum value of $|z^2+3z-1|$ in the disk $|z|\le 1$ **(5 Marks)**

QUESTION TWO: (20 MARKS)

a) Find the Laurent series expansion for $\dfrac{1}{z^2-3z+2}$ in the region $1<|z|<2$ **(7 Marks)**

b) Evaluate $\oint_c \dfrac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z-2)}dz$ where c is the circle $|z|=3$ **(5 Marks)**

c) Locate the zeros and singularities of the function $f(z)=\dfrac{(z^2-4)Cos\left(\dfrac{1}{z}\right)}{z^2+z-6}$

Classify the singularities and determine the behavior of the function at infinity **(8 Marks)**

QUESTION THREE: (20 MARKS)

a) State and prove Rouche's theorem **(6 Marks)**

b) Determine the number of roots of the polynomial $p(z)=z^4+6z-3$ that lie inside the ring $1\le|z|<2$ **(4 Marks)**

d) State and prove Cauchy's integral theorem **(5 Marks)**

e) Evaluate $\int_c \dfrac{dz}{(z^2+4)^2}$ where C is the circle $|z-i|=2$, using Cauchy's integral formula for derivatives. **(5 Marks)**

QUESTION FOUR: (20 MARKS)

a) Prove that if f(z) is analytic and $f'(z) \neq 0$ in a region R, then the mapping w = f(z) is conformal at all points in R. **(5 Marks)**

b) Find the bilinear transformation which maps the point 2, i, -2 onto the points 1, i, -1 **(5 Marks)**

c) Find the first four terms of the Taylor series expansion of $f(z) = \dfrac{1}{(z-1)(z-3)}$ about the point z = 4. Find the region of convergence. **(5 Marks)**

d) Find the image of the circle $|z-1| = 1$ under the mapping $w = \dfrac{1}{z}$ **(5 Marks)**

QUESTION FIVE: (20 MARKS)

a) Define the residue of a function and derive the formula for evaluating the residue at a pole of order $m > 1$ **(5 Marks)**

b) Apply Cauchy's Residue theorem to evaluate

(i) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^4+1}dx$ **(5 Marks)**

(ii) $\displaystyle\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle $|z| = \dfrac{3}{2}$ **(5 Marks)**

(iii) $\displaystyle\int_0^{2\pi} \frac{1}{4\sin\theta+5}d\theta$ **(5 Marks)**