2022

(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER MAIN EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS

COURSE CODE: **SPH 313**

COURSE TITLE: **QUANTUM MECHANICS I**

DATE: WEDNESDAY 20TH APRIL, 2022 **TIME:** 3:00 PM - 5:00 PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 Hours

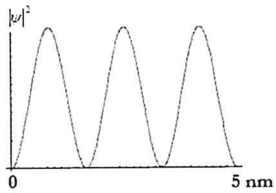
Answer question ONE and any TWO of the remaining.

Symbols used bear the usual meaning.

MMUST observes ZERO tolerance to examination cheating

QUESTION ONE [30 Marks]

(i) Consider an electron trapped in a 1D well with L = 5 nm. Suppose the electron is in the following state:



$$|\psi(x,t)|^2 = \frac{2}{L}\sin^2\left(\frac{n\pi x}{L}\right)$$

Assume that the potential seen by the electron is approximately that of an infinite square well. What is the energy of the electron in this state (in eV)? (mass of an electron $m_e = 9.11 \times 10^{-31}$ kg, Planck's constant, $h = 6.626 \times 10^{-34}$ J-s ;) [4 marks]

(ii) Consider an electron with an energy of 3.2×10^{-19} J impinging on a potential barrier with $V_0 = 3.2 \times 10^{-18}$ J and a width of 0.3 nm. Calculate the probability of an electron tunneling through the potential barrier (see figure 1). [4 marks]

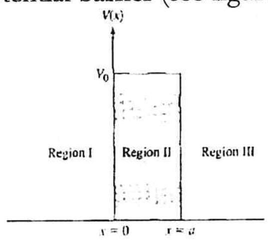


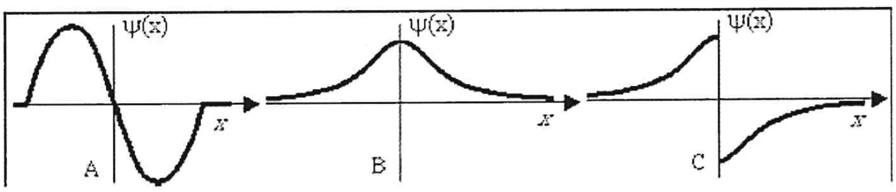
Figure 1 The potential barrier function

The probability that a particle impinging a potential barrier will penetrate the barrier and will appear in region III is given by

$$T \approx 16\left(\frac{E}{V_0}\right)\left(1 - \frac{E}{V_0}\right)e^{-2K_2 a} \quad ; \quad \text{where} \quad K_2 = \sqrt{\frac{2mE(V_0 - E)}{\hbar^2}}$$

(iii) Explain what is meant by the orthogonality of two wavefunctions $\psi_1(x)$ and $\psi_2(x)$, in the quantum mechanics of a particle on a line $-\infty < x < \infty$ [3 Marks]

(iv) Briefly explain giving reasons which of the following wavefunctions is a valid and which one is NOT a valid quantum mechanical wave function?? [3 marks]



(a) A (b) B (c) C

(v) The flux density, J_x, in the x-direction is defined as

$$J_x = \frac{\hbar}{2mi}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right).$$

Calculate the flux density for a system described by $\psi(x) = Ae^{ikx}$ and interpret your result. [4 marks]

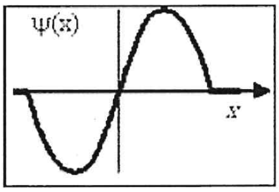
(vi) An electron is acted by a potential V(x) within the region $0 \le x \le a$. Its wavefunction is given as

$$\psi(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) e^{-i\omega t} \qquad 0 \le x \le a$$

Use the **time dependent** Schrodinger equation to show that the potential V(x) acting on the electron is given by [5 marks]

$$V = \frac{-\hbar^2 \pi^2}{2ma^2} + \hbar\omega$$

(vii) Consider the n = 2 state for the particle confined to an infinite square well as shown in the figure. Sketch a corresponding probability density function graph for the particle and hence state precisely where the particle is most likely to be found within the well. [3 marks]



(viii) Write down an expression for the energy of a particle confined in a two dimensional square well and use it to show that a two dimensional rectangular box with sides $L_1 = L$ and $L_2 = 2L$ there is an accidental degeneracy between the states $|n_1, n_2\rangle = |1, 4\rangle$ and $|n_1, n_2\rangle = |2, 2\rangle$ [4 marks]

QUESTION TWO [20 Marks]

Consider a particle of mass m which is confined to move freely in the one/dimensional interval $0 \le x \le L$. In other words, the potential V is given by V = 0 for $0 \le x \le L$ and V = ∞ for $x > L$ and $x < 0$.

(i) Write down the time-independent Schrodinger equation and the boundary conditions on the wavefunction. [3 marks]

(ii) Consider the wavefunction

$$\psi(x) = A\left[\sin\left(\frac{2\pi x}{L}\right) + 4\sin\left(\frac{6\pi x}{L}\right)\right].$$

Write $\psi(x)$ in terms of the normalised solutions to the Schrodinger equation

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), \qquad n = 1, 2, 3, ...$$

and determine the normalisation constant A. [6 marks]

(iii) Determine the expectation value of the energy in the state described by the wavefunction in part (ii). It is useful to remember that the value of energy E_n corresponding to $\psi_n(x)$ is $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ with n = 1, 2, 3, ….. [6 marks]

(iv) At the time t = 0 the energy is measured and is found to be equal to $\frac{18\pi^2\hbar^2}{L^2 m}$. Write down the normalised wavefunction after the measurement has taken place. Find the average value of x after the measurement has taken place. You may find the following indefinite integral useful [6 marks]

$$\int z \sin^2 z \, dz = \frac{z^2}{4} - \frac{\cos(2z)}{8} - \frac{z \sin(2z)}{4} + \text{constant}$$

QUESTION THREE

A beam of particles is incident from the left on a potential step given by

$$V(x) = \begin{cases} 0 & x < 0 \quad (\text{Region I}) \\ -V_0 & x \geq 0 \quad (\text{Region II}) \end{cases}$$

where $V_0 > 0$. The particle energy E is positive.

(a) Write down the time independent Schrödinger equation in both regions. Find its solutions and associated wavenumbers. [4 marks]

(b) State the boundary conditions that must be satisfied at $x = 0$ and apply them to the wavefunctions found in (a). [4 marks]

(c) Derive the expression for the probability that the incident particle will be reflected at the step. [5 marks]

(d) Hence find the values of the reflection probability in the limits of $V_0 \to 0$ and $V_0 << E$. [2 marks]

(e) Calculate the fluxes on both sides of the step and show that they are equal. [2marks]

(f) Why do they have to be equal ? [1mark]

(g) Sketch the wavefunction on both sides of the step. On which side of the step is the wavelength shorter ? [2 marks]

QUESTION FOUR [20 Marks]

At t = 0, a particle in a harmonic – oscillator potential is in the initial state

$$\psi(x,0) = \frac{1}{\sqrt{5}}\psi_1(x) + \frac{2}{5}\psi_2(x)$$

For a harmonic oscillator, $E_n = (n+1)\frac{\hbar\omega}{2}$

a) Calculate the expectation value of energy in the state $\psi(x,0)$ [7 marks]

b) Find the state of the particle $\psi(x, t)$ at a later time t. Is this state a stationary state ? [4 marks]

c) Calculate the expectation value of x for the state .$\psi(x, t)$ [5 marks]

d) What is the frequency of oscillation of this expectation value ? [4 marks]

QUESTION FIVE [20 MARKS]

a) Write down the time-independent Schrodinger equation for a particle in a one-dimensional harmonic oscillator potential, $V = \frac{m\omega^2 x^2}{2}$ [3 marks]

b) The ground-state wave function is of the form $\psi = A\,\exp(-\alpha x^2)$. Determine the constant α, and hence the ground-state energy. [10 marks]

c) A particle of mass m is confined to a harmonic oscillator potential given by $V = m\omega^2 x^2/2$, where $\omega = K/m$ and K is the force constant. The particle is in a state described by the wave function

$$\psi(x,t) = A\,e^{\left(\frac{-m x^2 \omega}{2\hbar} - \frac{i\omega t}{2}\right)}$$

Verify that this is a solution of Schrodinger's equation. [7 marks]