MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER MAIN EXAMINATIONS

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN PHYSICS AND BACHELOR OF EDUCATION SCIENCE

COURSE CODE: SPH 314

COURSE TITLE: ELECTROMAGNETISM

DATE:THURSDAY 28TH APRIL 2022 TIME: 8:00 AM–10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any TWO of the remaining
Symbols used bear the usual meaning.

MMUST observes ZERO tolerance to examination cheating

This Paper Consists of 5 Printed Pages. Please Turn Over.

You may need the following information

Mass of an electron, $m_e = 9.0 \times 10^{-31}$ kg

Mass of proton, $m_p = 1.67 \times 10^{-27} kg$

Charge of an electron, $q = -1.6 \times 10^{-19} C$

Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 / N.m^2

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ kgm/C^2

Speed of light, $c = 3 \times 10^8$ m/s

Electronic charge, $e = -1.6 \times 10^{-19}$ C

Divergence Theorem $\oint_s A.\, dS = \int_\tau \nabla . A\, d\tau$

Stokes Theorem $\oint_c A\,.\,dl = \int_s (\nabla \times A)\,.\,dS$

QUESTION ONE (COMPULSORY) [30 Marks]

(a) A proton is projected with a speed of $3 \times 10^6\ m/s$ horizontally from east to west. A uniform magnetic field $\vec{B}$ of strength $2.0 \times 10^{-3}\ T$ exists in the vertically upward direction.

i. Find the force on the proton just after it is projected. [2 marks]

ii. What is the acceleration produced? [1 marks]

(b) A current of 10.0 nA is established in a circular loop of radius 5.0 cm. Find the magnetic dipole moment of the current loop. [2 marks]

(c) A charge of 2.0 μC moves with a speed of $2.0 \times 10^{-6}\ m/s$ along the positive X-axis. A magnetic field $\vec{B}$ of strength $(0.20\vec{j} + 0.40\vec{k})T$ exists in space. Find the magnetic force acting on the charge. [3 marks]

(d) Figure below shows a triangular loop *PQR* carrying a current *i*. The triangle is equilateral with edge-length *l*. A uniform magnetic field *B* exists in a direction parallel to *PQ*. Find the forces acting on the three wires *PQ*, *QR* and *RP* separately. [4 marks]

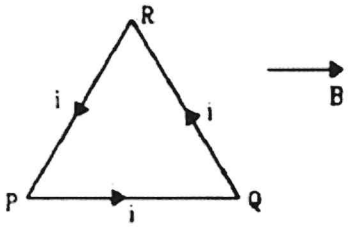

Figure 1.

(e) A singly charged magnesium (A=24) ions are accelerated to kinetic energy 2 keV and are projected perpendicularly into a magnetic field B of magnitude 0.6 T.

i) Find the radius of the circle formed by the ions. [3 marks]

ii) If there are also singly charged ions of the isotope magnesium 26, what would be the radius for these particles? [2 marks]

(f) Figure 2 shows two long, straight wires carrying electric currents in opposite directions. The separation between the wires is 5.0 cm. Find the magnetic field at a point P midway between the wires. [3 marks]

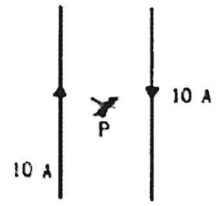

Figure 2.

(g) A bar magnet having a magnetic moment of $1.0 \times 10^4 J/T$ is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5} T$ exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field. [3 marks]

(h) A magnetic scalar potential due to a magnetic dipole at a point on its axis situated at a distance of 20 cm from its centre is found to be $1.2 \times 10^{-5}\ T - m$. Find the magnetic moment of the dipole. [4 marks]

(i) The maximum electric field in a plane electromagnetic wave is 600N/C. The wave is going in the x-direction and the electric field is in the y-direction. Find the maximum magnetic field in the wave and its direction. [3 marks]

QUESTION TWO (20 Marks)

(a) Show that the magnetic field due to current in a straight wire on a perpendicular bisector, is given by: [7 marks]

$$B = \frac{\mu_o i a}{2\pi d\sqrt{a^2 + 4d^2}}$$

(where $OP=d$, O is the foot of the perpendicular from P to the wire. a is the length of the wire)

(b) Show that

i) The magnetic scalar potential due to a Magnetic Dipole is given by; [6 marks]

$$V(r) = \frac{\mu_o}{4\pi}\frac{M\cos\theta}{r^2}$$

ii) The magnetic field due to a dipole is given by [7 marks]

$$B = \frac{\mu_o}{4\pi}\frac{M}{r^3}\sqrt{1 + 3cos^2\theta}$$

QUESTION THREE (20 Marks)

(a) An electron moves with a constant speed v along a circle of radius r.

i) Find the equivalent current through a point on its path. [2 marks]

ii) Find the magnetic moment of the circulating electron. [2 marks]

iii) Find the ratio of the magnetic moment to the angular momentum of the electron.
[2 marks]

(b) Figure 3 shows two long metal rails placed horizontally and parallel to each other at a separation l. A uniform magnetic field $\vec{B}$ exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.

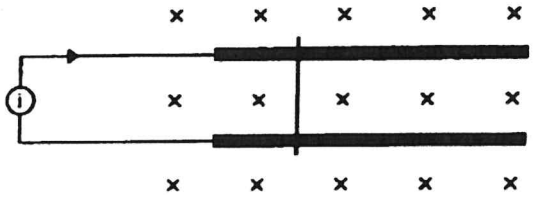

Figure 3.

i) What should be the minimum value of μ which can prevent the wire from sliding on the rails? [3 marks]

ii) Describe the motion of the wire if the value of μ is half the value found in b i) above.
[2 marks]

(c) The magnetic field B due to a current-carrying circular loop of radius 12 cm at its centre is $0.50 \times 10^{-4}T$. Find the magnetic field due to this loop at a point on the axis at a distance of 5.0 cm from the centre. [4 marks]

(d) A solenoid of length 10 cm and radius 1 cm contains 200 turns and carries a current of 10 A. Find the magnetic field at a point on the axis at a distance of 10 cm from the centre.
[5 marks]

QUESTION FOUR (20 Marks)

a) Given Faraday's law of electromagnetic induction, $V = -\frac{d\Phi}{dt}$, derive the equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$ [5 marks]

b) Describe the hysteresis loop and compare the hysteresis loop for a soft iron and steel
[6 marks]

c) A bar magnet of magnetic moment 2.0 A-m^2 is free to rotate about a vertical axis through its centre. The magnet is released from the east-west position. Find the kinetic energy of the magnet as it takes the north-south position. The horizontal component of the earth's magnetic field is $B = 25\mu T$. [4 marks]

d) A parallel-plate capacitor is being charged. Show that the displacement current across an area in the region between the plates and parallel to it as shown in figure 4, is equal to the conduction current in the connecting wires. [5 marks]

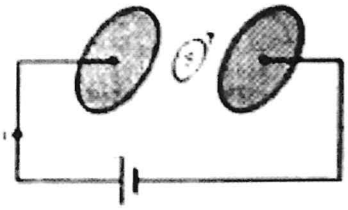

Figure 4.

QUESTION FIVE (20 Marks)

(a) An inductor (L = 20 mH), a resistor (R = 100 Ω) and a battery (ε = 10 V) are connected in series. Find

i. The time constant [1 marks]

ii. The maximum current [1 marks]

iii. The time elapsed before the current reaches 99% of the maximum value. [2 marks]

(b) Show that Ampere's circuital law $\oint \mathbf{B}.d\mathbf{L} = \mu_0 \sum I$, where B is the magnetic field vector, d**L** is the vector specifying an element of a closed path and I is the current, leads to the differential equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, where J is the current density. [4 marks]

(c) An infinitely long, cylindrical conductor of radius a carries total current I distributed uniformly a cross the conductor. Derive expressions for the magnetic field at a distance r from the centre of the conductor the cases (i) r< a and (ii) r> a. Sketch the variation of the field with r. [4 marks]

(d) State Maxwell's equations in terms of electric (**E**) and magnetic (**B**) fields in a region in which there are charge and current densities (***p*** and **J** respectively) which are functions both of space and time. For each equation briefly explain the physical concepts that it encapsulates. [5 marks]

(e) What form do the Maxwell equations reduce to in a vacuum i.e. in the absence of any dielectric or magnetic material? [3 marks]