2020

(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER MAIN EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS

COURSE CODE: SPH 416

COURSE TITLE: STATISTICAL MECHANICS

DATE: TUESDAY 26TH APRIL, 2022 **TIME:** 12:00 PM - 2:00 PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 Hours

Answer question ONE and any TWO of the remaining.
Symbols used bear the usual meaning.

MMUST observes ZERO tolerance to examination cheating

This Paper Consists of 4 Printed Pages. Please Turn Over.

YOU MAY USE THE FOLLOWING CONSTANTS

Atomic mass unit (u) = $1.6605 \times 10^{-27} kg$

Boltzmann constant $K = 1.38 \times 10^{-23} J/k$

Stefan's constant $\sigma = 5.67 \times 10^{-8} Watt/m^2k^4$

Gravitational acceleration of the earth = $9.8 m/s^2$.

Planck's constant h = $6.625 \times 10^{-34}\ JS$

Avogadro number $N_A = 6.023 \times 10^{23}$ mole^{-1}

Molar gas constant R = $8.3144 J/mole$

Mass of electron m = $9.1 \times 10^{-31} kg$

Speed of light $c = 3 \times 10^8 m/s$

Charge of electron $e = 1.6 \times 10^{-19} C$

QUESTION ONE (30 MARKS)

a) Define the term phase space. (1 mark)

b) Distinguish between Microcanonical and canonical ensembles as used in statistical mechanics. (4 marks)

c) Prove the relation (4 marks)

$$\overline{(E - \bar{E})^2} = K_B T^2 C_V$$

d) Consider a system of two free independent particles. Assuming that there are only two single particle energy levels $\in_1$, $\in_2$. By enumerating all possible two body microstates, determine the partition function Z2 if these two particles are:

i. Distinguishable (3 marks)

ii. Indistinguishable (3 marks)

iii. Determine the average energy of the two particle system (2 marks)

e) Derive the relation for the second law of thermodynamics (3 marks)

$$S = C_V \ln T + R \ln V + constant$$

f) A system consisting of N constituent particles of which can occupy four spin 3/2 quantum states with energies $E_1 = -\frac{3}{2}\varepsilon, E_2 = -\frac{1}{2}\varepsilon$, $E_3 = +\frac{1}{2}\varepsilon$, $E_4 = +\frac{3}{2}\varepsilon$ and multiplicities 1, 3, 3, 1 respectively. The system is in thermal equilibrium with reservoir at temperature T. Calculate

i. The Partition function Z_N of N particles which are distinguishable (3 marks)

ii. Helmholtz free energy (3marks)

iii. The entropy S (2 marks)

iv. The mean energy of the system (2 marks)

QUESTION TWO (20 MARKS)

a) A simple model for one dimensional solid consist of M independent oscillators each with energy (5 marks)

$$\in (x,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

where ω is the angular frequency? Determine the Partition function for M oscillators (distinguishable) and mean energy for the system.

b) Find for the probability distribution for a system in particular microstate r and has energy E_r in contact with heat reservoir at temperature T (canonical ensemble). (8 marks)

c) Derive entropy relation in terms of probability for a system in canonical ensemble (4 marks)

d) Assuming that only the ground state and the first excited state of a simple harmonic oscillator are appreciably populated, find the mean energy of the oscillator. (3 marks)

QUESTION THREE (20 MARKS)

a) Determine the mean occupation number $\bar{N}$ for Maxwell Boltzmann distribution. (8 marks)

b) Consider a system with only two accessible energy levels E_1 and E_2 and in thermal equilibrium with reservoir at temperature T. The lower level E_1 is non -degenerate while the upper level E_2 is two-fold degenerate.

i. Determine the probability that the lower level is occupied (2 marks)

ii. Calculate the mean energy (2 marks)

c) The partition function of a system is given by

$Z = (\exp[\beta T^3 V])^N$ where is a constant. Calculate the pressure P, the entropy S and mean energy of the system (6marks)

d) Prove the entropy relation for canonical ensemble (Symbols bear their usual meanings) $S = K[lnZ + \beta\bar{E}]$. (2 marks)

QUESTION FOUR (20 MARKS)

a) Obtain the curie law of paramagnetic on the basis of canonical ensemble. (10 marks)

b) A solid containing non interacting paramagnetic atoms each having a magnetic moment equal to Bohr magneton is placed in a magnetic induction field strength of 3 Tesla. Assuming that the atoms are in thermal equilibrium with the lattice, find the temperature at which the solid must be cooled so that 60% of the atoms are polarized with their magnetic moment parrallel to the magneic field. Take one Bohr magneton $\mu = 9.27 \times 10^{-24}$ J/T (6 marks)

c) Derive the relation for mean pressure $\bar{p}$ for a system in canonical ensemble in terms of partition function Z (4 marks)

QUESTION FIVE (20 MARKS)

a) What is blackbody radiation? Show that radiation pressure is equal to 1/3 of the energy density (10 marks)

b) Obtain the values of $\overline{N}$ and $\overline{N^2}$ in terms of grand partition function hence show that the fluctuations in the number of particles is given by $\overline{(\Delta N)^2} = \frac{1}{\beta}\frac{\partial \overline{N}}{\partial \mu}$ (8 marks)

c) Explain the equipartition theorem (2 marks)