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(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS 2021/2022 ACADEMIC YEAR

FIRST YAER SECOND SEMESTER MAIN EXAMINATIONS

FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

COURSE CODE: SPH 843E

COURSE TITLE: THEORY OF SUPERCONDUCTIVITY

DATE: MONDAY 25TH APRIL, 2022 **TIME:** 9:00 AM - 12:00 PM

INSTRUCTIONS TO CANDIDATES

TIME: 3 Hours

- **Answer any FIVE questions**
- **Symbols used bear the usual meaning.**

MMUST observes ZERO tolerance to examination cheating

QUESTION ONE (14 MARKS)

Under high frequency fields, the superconducting Cooper pairs are also accelerated since they have to change direction on reversal of fields. Starting with Maxwell's equation; $\nabla \mathbf{H} = \mathbf{J_n} + \mathbf{J_s} + \frac{d\mathfrak{D}}{dt}$ and knowing that $\dot{\mathbf{J}}_\mathbf{s} = \frac{E}{\mu\lambda^2}$, show that,

(a). $\nabla^2\mathbf{H} = \left[\frac{1}{\lambda^2} - \omega^2\mu_0\mathbf{E} + (i\omega\mu_0\sigma_0)\right]\mathbf{H}$ (10 marks)

(b). For d.c fields, show that the equation in (a) reduces to; $\nabla^2\mathbf{H} = \frac{\mathbf{H}}{\lambda^2}$ (2 marks)

(c). At microwave frequencies show that the equation in (a) becomes;

$$\nabla^2\mathbf{H} = \left[\frac{1}{\lambda^2} + (i\omega\mu_0\sigma_0)\right]\mathbf{H} \text{ (2marks)}$$

QUESTION TWO (14 MARKS)

(a). The difference in free energy between the normal and the superconducting state in applied field of strength H_a is given by; $g_n + g_s(\mathbf{H_a}) = \frac{1}{2}\mu_0(H_c^2 - H_a^2)$, where g is free energy density.. If Gibb's free energy of a magnetized material is $G = U - TS + PV - \mu_0 H_a M$, show that

(i) the change in entropy when the material transits from the normal to the superconducting state is; $s_n - s_s = \mu_0\mathbf{H_c}\frac{d\mathbf{H_c}}{dT}$ (7 marks)

(ii) From (a), deduce that the degree of order in the superconducting state is much higher than that of the normal state. (3 marks)

(b). Derive the London's equations. (4 marks)

QUESTION THREE (14 MARKS)

(a). Briefly describe the working of a superconducting fuse and circuit breaker. (2 mark)

(b). The Hamiltonian which represents the interaction between two electrons mediated through a phonon exchange is given as;

$$H_I = \sum_q \sum_{k,k'} \left|B_q\right|^2 \frac{\hbar\omega_q}{(\varepsilon_k - \varepsilon_{k-q})^2 - \hbar^2\omega_q^2} C_{K'+q}^{+} C_{k-q}^{+} C_k C_{K'}$$

where k and k' are the wave vectors of the electrons and q is phonon wavevector.

(i) Write down the electron anti-commutation relations that the operators, Cs obey (4 marks)

(ii) Fully describe the quantity B_q and state the condition for an attractive electron-electron interaction.(5 marks)

(c). Describe any 3 salient features of the BCS theory of superconductivity. (3 marks)

QUESTION FOUR (14 MARKS)

(a). Bogoliubov- Valatin canonical transformation allows us to write the H_{BCS} in terms of new operators. By defining the new operators as;

$\gamma_k = U_k C_k - V_k C_{-k}^{+}$ and

$\gamma_{-k} = U_k C_{-k} + V_k C_k^{+}$ Fully diagonalize the H_{BCS} (8marks)

(b). Obtain the kinetic and the potential energy of a superconducting system whose trial wave function is described as; $|\Psi_0\rangle = \prod_k \left[(1 - P_k)^{\frac{1}{2}} + P_k^{\frac{1}{2}} b_k^*\right] |\Psi_0\rangle$ where P_k is the probability of pair occupation. (6 marks)

QUESTION FIVE (14 MARKS)

(a). Discuss the two characteristic lengths of a superconductor (4 marks).

(b). By calculating the integral of the phase of the Cooper pairs around a closed loop and using Stoke's theorem show that the magnetic flux is given by $\phi = \frac{2\pi\hbar}{2e}(n)$ where n is an integer. (10 marks)

QUESTION SIX (14 MARKS)

In Josephson junction, $\psi_1 = n_1^{\frac{1}{2}} e^{i\theta_1}$ is the probability amplitude of the electron pairs on one side of the junction and $\psi_2 = n_2^{\frac{1}{2}} e^{i\theta_2}$ is the amplitude on the other side. Suppose both superconductors are identical and at zero potential, the Schrodinger equation of such a system can be; $i\Box \frac{\partial \psi}{\partial t} = T\psi$, where T is a measure of the leakage of ψ_1 into the region 2 and ψ_2 into the region 1. Show that;

(a). $\frac{\partial n_2}{\partial t} = \frac{\partial n_1}{\partial t}$ (12 marks)

(b). The supercurrent **J** is $J = J_0 \sin\delta$, where δ is phase difference. (2marks)