(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

(MMUST)

DEPARTMENT OF INFORMATION TECHNOLOGY

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

(MAIN EXAMINATION)

SECOND YEAR FIRST SEMESTER EXAMINATION

FOR THE

DIPLOMA   IN

INFORMATION TECHNOLOGY

COURSE CODE: DIT 071

COURSE TITLE: DISCRETE MATHEMATICS

DATE: 22/04/2022   **TIME:**   8:00am – 10:00am

INSTRUCTIONS TO CANDIDATES

- *Answer question ONE and ANY OTHER TWO questions.*
Time 2 Hours

MMUST observes ZERO tolerance to examination cheating

QUESTION ONE - COMPULSORY   [24 MARKS]

a) Define the following terms
   i) Function   (2 marks)
   ii) Set   (1 mark)
   iii) Binary tree   (2 marks)

b) Assume that in a class of 800 students, 300 students are taking a mathematic course, 400 are taking a course in physics and 100 are taking both mathematics and physics. How many students are taking at least one of those courses? Draw a Venn diagram to represent this information   [4 marks]

c) Show that $(p \rightarrow q) \rightarrow (\neg p \vee q)$ is a tautology   [4 marks]

d) Given the sets   A= {0, 1, 2, 3, 4} B= {3, 4, 5, 6}   C = {2, 3, 5}   and the universal set Ω = {x : 0 $\leq$ x $\leq$ 10}. Prove that;
   i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (4 marks)
   ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (4 marks)

e) Given the functions $f(x) = x^2 + 2x + 5$ and $g(x) = 2x + 6$. Find
   i. $f \circ g(-2)$ (3 marks)
   ii. $g^{-1}(x)$ (2 marks)

QUESTION TWO   [18 MARKS]

a) Define the following

   i)   A predicate.   (2 marks)
   ii) A theorem   (2 marks)
   iii) A proof   (2 marks)

b) Proof by contradiction that if $x + y > 5$ then either $x > 2$ or $y > 3$ (4 marks)

c) Assume that in a class of 1000 students, 200 students are taking a mathematic course, 300 are taking a course in physics and 100 are taking both mathematics and physics. How many students are taking at least one of those courses? Draw a Venn diagram to represent this information   (4 marks)

d) Define

   i)   Planar graph   (2 marks)
   ii) Rooted tree   (2 marks)

QUESTION THREE [18 MARKS]

a) Show that $(p \rightarrow q) \rightarrow (\neg p \vee q)$ is a tautology [4 marks]

b) Given the function $f(x) = \frac{x+3}{x-1}$. Find $f^{-1}(x)$ hence show that [5 marks]

$$f \circ f^{-1}(x) = x$$

c) Use the truth tables to prove that $\neg(r \vee s) \equiv \neg r \wedge \neg s$ (4 marks)

d) Let M, P and C be the sets of students taking mathematics, Physics and Computer courses respectively in a college. Assume that $|M| = 300$, $|P| = 350$, $|C| = 450$, $|M \cap P| = 100$, $|M \cap C| = 150$, $|P \cap C| = 75$ and $|M \cap P \cap C| = 10$

  i. Represent the information on a Venn diagram. (3 marks)
  ii. How many students are taking at least one the courses? (2 marks)

QUESTION FOUR (18 MARKS)

a) Differentiate between a Simple graph and a Multigraph (2 Marks)

b) Let A and B be two sentences A: "Jack is a boy" and B: "Jill is a girl" Express the following symbolic form to statement

  i. $A \wedge B$ (1 marks)
  ii. $A \rightarrow B$ (1 mark)

c) Show that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ is logically equivalent. [4 marks]

d) Show that $(p \wedge \neg q) \wedge (\neg p \vee q)$ is a contradiction. [5 marks]

e) Represent on a Venn diagram $A \cap B$, $\overline{A \cap B}$ and $\bar{A} \cap B$ (4 marks)

QUESTION FIVE [18 MARKS]

a) Show that the following propositions are logically equivalent
  i) $\neg(p \leftrightarrow q)$ and $p \oplus q$ (4 marks)

ii) Hence show that $\neg(p \leftrightarrow q) \leftrightarrow (p \oplus q)$ is a tautology (2 marks)

b) Proof by contradiction that if $x + y > 9$ then either $x > 5$ or $y > 4$ (4 marks)

c) If $A = \{a, b, c, d\}$ and $B = \{c, d, e, f, g\}$. Show that $A - (A - B) = A \cap B$ (3 marks)

d) Proof by contradiction that $\sqrt{2}$ is not a rational number [5 marks]