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(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

(MAIN EXAMINATION)

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE (ECT, ETM)

COURSE CODE: **MAT 213**

COURSE TITLE: **LINEAR ALGEBRA I**

DATE: 27/04/2022 TIME: 12.00-2.00PM

Instructions to candidates:
Answers question ONE (COMPULSORY) and any other TWO questions.

Time: 2 hours

QUESTION ONE (30MARKS) COMPULSORY

a) Consider the set of real 2×2 matrices M_2 .Show that M_2 is a vector space. (4marks)

b) Determine the rank of the matrix $A = \begin{pmatrix} 3 & -3 & 3 \\ 2 & -1 & 4 \\ 3 & -5 & -1 \end{pmatrix}$ (3marks)

c) Show that the system below has no solution (4marks)

$$x_1 + x_2 + 5x_3 = 3$$
$$x_2 + 3x_3 = -1$$
$$-x_1 + 2x_2 + 8x_3 = 3$$

d) i) Determine whether the vector (8,0,5) is a linear combination of vectors $(1,2,3), (0,1,4)$ *and* $(2,\text{-}1,1)$ in $\mathbb{R}^n$ (3marks)

ii) Let $f(x) = 2x^2 - 5$ and $g(x) = x + 1$. Show that the function $h(x) = 4x^2 + 3x - 7$ lies in the subspace span $\{f, g\}$ in set of polynomials of degree 2. (4marks)

e) i) Distinguish between linear dependence and independence of vectors.

ii) Show that the set $\{x^2 + 1, 3x - 1, -4x + 1\}$ is linearly independent in P_2. (3marks)

f) i) Define a linear transformation from a vector space U into vector space V. (2marks)

ii) Determine if $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defined as $T(x_1, x_2) = (2x_1, 2x_2)$ is a linear transformation (5marks)

QUESTION TWO (20 MARKS)

a) Let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be defined by $T(x) = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. Find

i) Nullity of T (6marks)

ii) Rank of T (3marks)

b) Given that $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear transformation defined by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 \\ -x_1 - x_2 \end{pmatrix}$. Find the matrix of T with respect to the standard basis. (4marks)

c) Show that if V is a vector space then any set of vectors in V that contains the zero vector is linearly independent. (3marks)

d) Show that the set $\{x + 1, x - 1, -x + 5\}$ is linearly independent in P_1 (4marks)

QUESTION THREE (20 MARKS)

a) Use Gauss-Jordan elimination to solve the system. (6marks)

$$4x_1 + 8x_2 - 12x_3 = 44$$
$$3x_1 + 6x_2 - 8x_3 = 32$$
$$-2x_1 - x_2 = -7$$

b) Use the row reduction method to obtain the inverse of the matrix (5marks)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$$

c) Find the determinant of the matrix $D = \begin{pmatrix} 1 & -2 & 0 \\ -3 & 5 & 1 \\ 4 & -3 & 2 \end{pmatrix}$ (3marks)

d) Solve, if possible the system of equation (6marks)

$$3x_1 - 3x_2 + 3x_3 = 9$$
$$2x_1 - x_2 + 4x_3 = 7$$
$$3x_1 - 5x_2 - x_3 = 7$$

QUESTION FOUR (20 MARKS)

a) Let P_n denote the set of real polynomial functions of degree$\leq n$. Prove that P_n is a vector space if addition and scalar multiplication are defined on the polynomials in a point-wise manner. (6marks)

b) Show that a set consisting of two or more vectors in a vector space is linearly independent if and only if it is possible to express one of the vectors as a linear combination of other vectors. (5marks)

c) Let the set $\{v_1, v_2\}$ be linearly independent. Prove that $\{v_1 + v_2, v_1 - v_2\}$ is linearly independent. (5marks)

d) Illustrate the addition axioms of a vector space V in $\mathbb{R}^n$ (4marks)

QUESTION FIVE (20 MARKS)

a) Use the method of Gauss-Jordan elimination to find the reduced echelon form of the system of equations. (5marks)

$$2x_3 - 2x_4 + 2x_5 = 0$$
$$3x_1 + 3x_2 - 3x_3 + 9x_4 + 12x_5 = 0$$
$$4x_1 + 4x_2 - 2x_3 + 11x_4 + 12x_5 = 0$$

b) If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 1 & 0 & 3 \end{pmatrix}$, find A^{-1} by use of determinants and cofactors of A. (8marks)

c) Solve by Cramer's rule the following set of simultaneous equations. (7marks)

$$5x - 6y + 4z = 15$$
$$7x + 4y - 3z = 19$$
$$2x + y + 6z = 46$$