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(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS 2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER EXAMINATIONS MAIN EXAM

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ENGINEERING TECHNOLOGY

COURSE CODE: **MAT 402**

COURSE TITLE: **Partial Differential Equations I**

DATE: **27TH APRIL 2022** **TIME:** **8.00-10.00 AM**

INSTRUCTIONS TO CANDIDATES

- Answer question ONE (COMPULSORY) and any other TWO questions

This Paper Consists of 5 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Classify the following PDEs (2 Marks)

(i) $\frac{\partial^2 u}{\partial x^2} + xy\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = 0$

(ii) $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2}\right)^4$

b) Solve the Lagrange equation $yzp + yzq = xy$ (4 Marks)

c) Find the integral surface of the equation

$$(x-y)p + (y-x-z)q = z$$

passing through the circle $z = 1,\ x^2 + y^2 = 1$ (5 Marks)

d) Find a PDE by eliminating the arbitrary function from the following equation

$$\phi(x+y+z,\ x^2+y^2-z^2) = 0$$ (5 Marks)

e) Using the Charpit's method, solve $p(q^2+1) + (b-z)q = 0$ (4 Marks)

f) Show that the equations

$$f = xp - yq - x = 0$$
$$g = x^2 p + q - xz = 0$$

Are compatible and find a one parameter family of common solutions (5 Marks)

g) A vibration string is governed by the equation

$$\frac{\partial^2 y}{\partial x^2} = c\frac{\partial^2 y}{\partial x^2}$$

If the string is fixed at two points L apart and is stretched. Find the displacement of any point at a distance x from one end at time t. (5 Marks)

QUESTION TWO (20 MARKS)

a) Form the PDE by eliminating the arbitrary constants a, b, c from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ (5 Marks)

b) Solve $(D^2 - 2DD' + D'^2)z = x^2 y^2 e^{x+y}$ (6 Marks)

c) Show that the following Pfaffian Differential Equation is integrable and hence find its integral

$$ydx + xdy + 2zdz = 0$$ (4 Marks)

d) Solve the PDE

$$p(1-q^2) = q(1-z)$$ (5 Marks)

QUESTION THREE (20 MARKS)

a) Solve $\left(\frac{y^2 z}{x}\right)p + zxq = y^2$ (5 Marks)

b) Verify that $u(x,t) = e^{-kt}\sin x$ satisfies the equation $u_t = ku_{xx}$ (4 Marks)

c) Find the surface where tangent plane cut off an intercept of constant length K from the axis z. (6 Marks)

d) Find the envelope of the system of surface generated by the PDE

$$(x-a)^2 + (y-b)^2 + z^2 = 1$$

Where a,b are parameters (5 Marks)

QUESTION FOUR (20 MARKS)

a) Find the integral surface of the PDE

$$(x-y)p + (y-x-z)q = z$$

through the circle $z = 1, x^2 + y^2 = 1$ (6 Marks)

b) Find the particular integral of the PDE (4 Marks)

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$$

c) Solve $q - p + x - y = 0$ (4 Marks)

d) Show that the equations $xp = yq$ *and* $z(xp + yq) = 2xy$ are compatible and hence solve it. (6 Marks)

QUESTION FIVE (20 MARKS)

a) Derive the PDE arising from the given equation $z = ax^b y^{1-b}$ (3 Marks)

b) Using the Lagrange's method, solve the PDE $\frac{y-z}{yz}p + \frac{z-x}{zx}q = \frac{x-y}{xy}$ (5 Marks)

c) Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ subject to the condition $z(x,0) = x^2$ and $z(1,y) = \cos y$ (6 Marks)

d) Consider the simple problem of heat conduction in a bar whose ends are held at zero temperature

$$u_t = ku_{xx}$$
$$u(0,t) = 0 = u(L,t)$$

Using the method of separation of variable, find the temperature distribution in the bar (6 Marks)