160

MASINDE MULIRO UNIVERSITY OF

SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

MAIN CAMPUS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ENGINEERING (MIE, CSE, ECE)

COURSE CODE: MAT 302/310

COURSE TITLE: DIFFERENTIAL EQUATIONS

DATE: 27th April, 2022 **TIME: 8.00-10.00 AM**

INSTRUCTIONS TO CANDIDATES

- Answer question ONE and **ANY OTHER TWO (2)** questions

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Classify the following differential equations (2 Marks)

(i) $\left(\frac{dy}{dx}\right)^2 + 2\frac{d^2y}{dx^2} = \cos 2x$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x^2 + y$

b) Solve the differential equation $\sqrt{y}dx + (1+x)dy = 0 \quad y(1) = 1$ (4 Marks)

c) By elimination of arbitrary constants, form a PDE from the equation (4 Marks)

$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

d) Show that equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and hence solve it (5 Marks)

e) A body where temperature T is initially 200°C is immersed in a liquid when temperature T_0 is constantly 160°C. If the temperature of the body is 150°C at t=1min, what is its temperature at t=2min. (5 Marks)

f) Find the integral surface of the linear PDE $x(y^2 + z^2)p - y(x^2 + 1)q = z(x^2 + y^2)$ which contains the straight line $x + y = 0,\ z = 1$. (5 Marks)

g) Using the method of undetermined coefficients, solve the following differential equation (5 Marks)

$$y'' - 4y = 2x$$

QUESTION TWO (20 MARKS)

a) Examine whether the following differential equation is homogeneous and hence solve it (5 Marks)

$(xy + y^2 + x^2)dx - x^2 dy = 0$

a) 20 grams of salt are dissolved into a beaker containing one litre of a certain chemical. The mass of salt, M grams, which remains undissolved t seconds later is modeled by the differential equation $\frac{dM}{dt} + \frac{2M}{20-t} + 1 = 0$, show clearly that $M = 0.1(10-t)(20-t)$ (5 Marks)

b) Using the method of variation of parameters, solve the following differential equation (5 Marks)

$$\frac{d^2y}{dx^2} + y = \tan x$$

b) Solve the differential equation $y''' - 4y' - 5y = 0 \quad y(0) = 1 \quad y'(0) = 2$ (4 Marks)

QUESTION THREE (20 MARKS)

a) Verify that the given differential equation

$$xU_{xx} - (2x+1)U_x - U_y + (x+1)U + \frac{2U}{y} = 0$$

is satisfied by the function $U = 3x^2y^2e^x$ (5 Marks)

b) By method of separation of variables, solve the following pde (5 Marks)

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

c) Using Charpit's method, find the complete integral of the pde (5 Marks)

$$2xz - px^2 - 2qxy + pq = 0$$

d) The voltage V and the current I at a distance x from the sending end of the transmission line satisfy the equations

$$-\frac{dV}{dx} = Ri \qquad -\frac{di}{dx} = GV$$

Where R and G are constants. If $V = V_0$ at the sending end ($x=0$) and $V=0$ at the receiving end ($x=l$). Show that $V = V_0\left\{\frac{\sinh n(l-x)}{\sinh nl}\right\}$ when $n^2 = RG$ (5 Marks)

QUESTION FOUR (20 MARKS)

a) Eliminate the arbitrary function from the equation (5 Marks)

$$x + y + z = f(x^2 + y + z^2)$$

b) Solve $(y+z)p + (z+x)q = (x+y)$ (5 Marks

c) Solve $p(q^2+1) + (b-z)q = 0$ using charpit's method (5 Marks)

d) A string is fixed at two points l apart and is stretched. The motion takes place by displacing the string in the form $y = \mathrm{a}\sin\frac{\pi x}{l}$ from which its released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by

$y(x,t) = \mathrm{a}\sin\frac{\pi x}{l}\cos\frac{\pi x}{l}$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Using the method of separation of variables, solve the heat equation given by (4 Marks)

$u_t - u_{xx} = 0$

b) Solve $(D^2 + 4DD' - 5D'^2)Z = xy + \sin(2x + 3y)$ (5 Marks)

c) It takes two years for three grams of a radioisotope to decay to 0.9g. Determine;

(i) The half life of the isotope (3 Marks)

(ii) The time it will take the amount to decay to 0.4 g (3 Marks)

d) Solve the differential equation $y'' - 6y + 13y = 0 \quad y(0) = 1 \quad y'(0) = 2$ (5 Marks)