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(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS SPECIAL/SUPPLEMENTARY EXAMINATIONS FOR 2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE (COM)

COURSE CODE: BCS 121

COURSE TITLE: DISCRETE STRUCTURES II

DATE: **MONDAY 01-08-2022** **TIME:** **8:00A.M-10:00A.M**

Instructions to candidates:
Answer Question one and any other two questions.

Time: 2 hours

This paper consists of 4 printed pages. Please turn

QUESTION ONE (30 MARKS)

a) Find a closed-form formula for the Fibonacci sequence defined by

$$F_{n+1} = F_n + F_{n-1} \ (n > 0);\ F_0 = 0, F_1 = 1.$$

(6 Marks)

b) Define the following terms

i) Recurrence relation

ii) Multigraph (4 Marks)

c) Find the probability of obtaining a sum of 10 after rolling two fair dice. (5 Marks)

d) Let

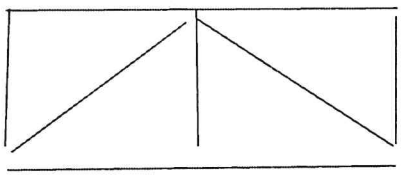
$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};\ B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

Find

i) $A \vee B$ (3 Marks)

ii) $A \wedge B$ (3 Marks)

e) Solve the Chinese postman problem for the following graph (6 Marks)



f) Let $P(x): x + 2$. Write formally the following statements and deduce there meaning:

i) "There is at least one integer x such that $x + 2 = 5$" (2 Marks)

ii) "for all x, $x + 2 = 5$" (2 Marks)

QUESTION TWO (20 MARKS)

a) Draw a binary search tree for the set $S = \{1,2,3,4,5,6,7,8,9,10\}$. (5 Marks)

b) Explain the Beadth- First search Algorithm. (9 Marks)

c) Define the following terms

i) Rooted tree

ii) Binary tree

iii) Decision tree (6 Marks)

QUESTION THREE (20 MARKS)

a) Assume that in a country with currently 100million people has a population growth rate of 1% per year and it also receives a hundred thousand immigrants per year. Find its population in 10 years. (6 Marks)

b) Convert these adjacency matrices into incidence matrices (6 Marks)

i) $\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$

ii) $\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$

c) A fair coin is tossed three times yielding the equiprobable space

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Consider the three events

$$A = \{First\ toss\ is\ heads\} = \{HHH, HHT, HTH, HTT\}$$
$$B = \{Second\ tos\ is\ heads\} = \{HHH, HHT, THH, THT\}$$
$$C = \{Exactly\ two\ heads\ in\ a\ row\} = \{HHT, THH\}$$

Show that A and B and A and C are independent, but B and C are dependent. (8 Marks)

QUESTION FOUR (20 MARKS)

a) Draw the graph with vertices A, B, C, D, E and edges BD, BC, CE, DE. (5 Marks)

b) Find the degree of each vertex in part (a) (5 Marks)

c) Find the incidence matrix of the graph in part (a). (5 Marks)

d) What is the solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$? (5 Marks)

QUESTION FIVE (20 MARKS)

a) Translate into English the predicate logic used in artificial language $\exists x(P(x) \Lambda \neg Q(x))$, where the P(x) says that x is a bird and Q(x) says that x can fly. Take the universe of discourse to be all living creatures. Suggest a witness to the existential quantifier (that is, a creature that makes the sentence true). (2 marks)

b) (i) Write in math notation the following English sentence: "Every number is divisible by 2 or by 3" (use d | n for "n is divisible by d"). (4 marks)

(ii) For which universe of discourse is it true? (1 mark)

(iii) For which universe of discourse is it false? (1 mark)

(iv) State it is true or false if the universe of discourse complex numbers (1 mark)

c) Assign symbols to represent the predicates and write the following in symbolic form supplying the domain of discourse. "**Every political party has its years**" (5 marks)

d) Form a binary search tree for the data 16, 24, 7, 5, 8, 20, 40 and 3 in the given order (6 marks)