



(University of Choice)

**MASINDE MULIRO UNIVERSITY OF  
SCIENCE AND TECHNOLOGY  
(MMUST)**

**MAIN CAMPUS**

**SUPPLEMENTARY/SPECIAL UNIVERSITY EXAMINATIONS**

**2021/2022 ACADEMIC YEAR**

**FIFTH YEAR FIRST SEMESTER EXAMINATIONS**

**FOR THE DEGREE  
OF  
BACHELOR OF SCIENCE IN ELECTRICAL  
AND  
COMMUNICATION ENGINEERING**

**COURSE CODE: ECE 513**

**COURSE TITLE: NON-LINEAR AND MULTIVARIABLE  
CONTROL**

**DATE: OCTOBER 3<sup>RD</sup>, 2022**

**TIME: 3:00PM - 5:00PM**

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**INSTRUCTIONS TO CANDIDATES**

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS.  
QUESTION ONE CARRIES 30 MARKS AND ALL OTHERS 20 MARKS EACH.

TIME: 2 Hours

MMUST observes ZERO tolerance to examination cheating

This Paper Consists of 4 Printed Pages. Please Turn Over. 

### QUESTION ONE (COMPULSORY) (30 MARKS)

1. Differentiate the following types of nonlinearities using appropriate examples.
  - i. Inherent nonlinearities and intentional nonlinearities
  - ii. Static nonlinearities and dynamic nonlinearities
  - iii. Functional nonlinearities and piece-wise nonlinearities.

[6 Marks]

2. State at least 4 differences between linear and nonlinear systems.

[4 Marks]

3. A 2<sup>nd</sup> order system is represented by  $\dot{x} = Ax$ . Where,  $A = \begin{bmatrix} -1 & 1 \\ -2 & -4 \end{bmatrix}$ , using Lyapunov theorems determine the stability of the system at the origin.

[4 Marks]

4. Derive the describing function of a simple dead zone

[8 Marks]

5. The response of a system is  $y = ax^3 + e^{bx}$ . Test whether the system is linear or nonlinear.

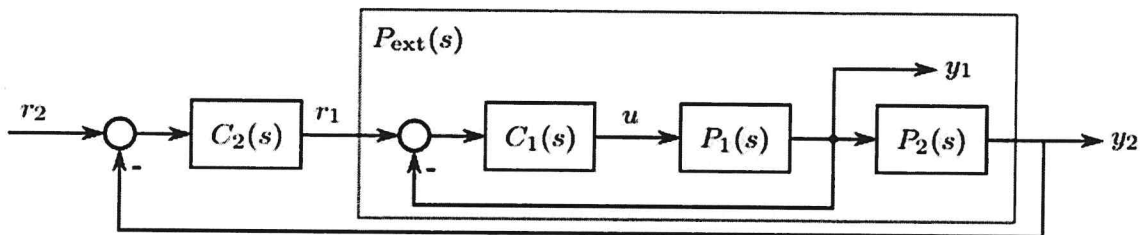
[4 Marks]

6. Discuss the terms stability in the large and stability in the small

[4 Marks]

### QUESTION TWO (20 MARKS)

1. The control system of a SISO system is given below



Where the plant  $P_1$  is given as  $P_1 = \frac{1}{s+1}$  and the inner controller  $C_1$  is given as  $C_1(s) = 5$ . To design the outer controller  $C_2$ , calculate the transfer function of the extended plant  $P_{ext}$  which includes the inner control loop and  $P_2$ , where  $P_2 = \frac{1}{5s+1}$ .

[4 Marks]

2. A nonlinear second order servo is described by the equation below

$$\ddot{e} + 2\zeta\omega_n\dot{e} + 2\omega_n e + e^2 = 0$$

Where  $\zeta = 0.25$ ,  $\omega_n = 1$  rad/sec.

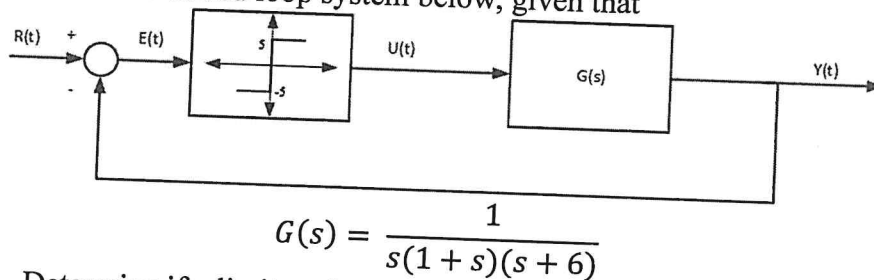
- i. Find all the singularities of the system
- ii. Classify all singularities
- iii. Sketch the phase portrait in the neighborhood of the equilibrium points

[10 Marks]

3. What are the desirable characteristics of the nonlinear element while performing a describing function analysis? [4 Marks]
4. State at least two disadvantages of linearization to solve non-linear systems. [2 Marks]

### QUESTION THREE (20 MARKS)

1. State the Aizerman's and Kalman's conjecture. [4 Marks]
2. Using phase plane diagrams, differentiate between the following: [4 Marks]
- Stable node and unstable node
  - Stable focus and unstable focus
  - Saddle point and vortex
3. Consider a nonlinear closed loop system below, given that [6 Marks]



- Determine if a limit cycle exist
- if so determine if the limit cycle is a sustained oscillation
- find the amplitude and frequency of the limit cycle.

[10 Marks]

### QUESTION FOUR (20 MARKS)

1. Consider a nonlinear system given by  $\ddot{y} + \dot{y} + y = 0$ . Construct the phase trajectory, using the method of Isoclines. Choose slope as  $N = \{-4, -3, -2, -1, 0, 1, 2, 3\}$ . [8 Marks]
2. Discuss stability with reference to Linear Time Invariant systems [2 Marks]
3. Given a scalar function  $V(x) = 4x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + x_2x_3 + 2x_1x_3$  represent it in quadratic form and based on Sylvester's Theorem determine its definiteness. [6 Marks]
4. State and explain Popov's hyperstability theorem. [4 Marks]

### QUESTION FIVE (20 MARKS)

1. An input  $x(t)$  and an output  $y(t)$  of a nonlinear system are related through a nonlinear differential equation. Find the describing function of the system.

$$y = x^2 \frac{dx}{dt} + 2x$$

[6 Marks]

2. Discuss the following terms using appropriate equations and drawings

- i. Lyapunov stability
- ii. Asymptotic stability
- iii. Quasi-asymptotic stability

**[6 Marks]**

3. For the system described by the equation below, determine the equilibrium point and check for its stability using Lyapunov's stability theorem.

$$\begin{aligned}\dot{x}_1 &= -x_1 - x_2(x_1^2 + x_2^2) \\ \dot{x}_2 &= x_2 - x_1(x_1^2 + x_2^2)\end{aligned}$$

**[8 Marks]**