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(The *University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST) MAIN CAMPUS

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE

OF

BACHELOR OF SCIENCE (SIK)

COURSE CODE: **BIK 111**

COURSE TITLE: **MATHEMATICS FOR INTELLIGENCE SYSTEMS**

DATE: 15/12/2022 **TIME: 8.00AM – 10.00AM**

INSTRUCTIONS TO CANDIDATES

Answer question ONE (compulsory) and any **TWO** questions

This Paper Consists of 4 Printed Pages. Please Turn Over

QUESTION ONE (30MKS)

a) Show that the $\lim_{n \to \infty} \frac{n^2+2}{2n^2-1} = \frac{1}{2}$. (4 Marks)

b) Evaluate the derivative of $f(x) = \sin x$ using first principles. (5 Marks)

c) Assume that in a university with 1000 students, 200 students are taking a course in mathematics, 300 are taking a course in physics, and 50 students are taking both. How many students are taking at least one of those courses? (5 Marks)

d) Define the following terms as used in graph theory
   i) Simple graph
   ii) Weighted graph
   iii) Hamilton circuit (6 Marks)

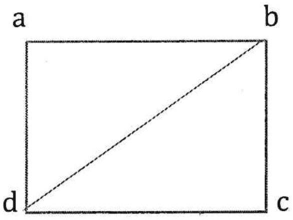
e) Let P and Q be propositions. Construct a truth table for the proposition $(P \wedge Q) \Rightarrow (P \vee Q)$. State and explain whether the statement is a tautology or a contradiction. (5 Marks)

f) Find $\int \log x \, dx$. (5 Marks)

QUESTION TWO (20MKS)

a) Write the negation 0f the following quantified statements
   i) $\neg[(\forall x)P(x)$
   ii) $\neg[All\ birds\ can\ fly]$ (4 Marks)

b) Find the adjacency matrix $A = [a_{ij}]$of the graph below (4 Marks)



c) Draw all trees with exactly six vertices (6 Marks)

d) Use the Midpoint Rule with $n = 5$ to approximate $\int_1^2 \frac{1}{x} dx$. (6 Marks)

QUESTION THREE (20MKS)

a) Determine

$$\int_0^3 (x^3 - 6x)\, dx$$ (6 Marks)

b) Define the following terms
   i) Continuous function
   ii) Derivative (4 Marks)

c) Let $f(x) = x^x$ for $x > 0$ and let $f(0) = 1$. Show that $f(x)$ is continuous at $x = 1$. (6 Marks)

d) Compute the limit (4 Marks)

$$\lim_{x \to 2} \frac{3x - 5}{x^2 - 1}$$

QUESTION FOUR (20MKS)

a) Given $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let R be the following relation from A to B:

$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$$

i) Determine the matrix of the relation. (4 Marks)

ii) Draw the arrow diagram of R. (3 Marks)

iii) Find the inverse relation R−1 of R. (3 Marks)

iv) Determine the domain and range of R. (2 Marks)

b) Given $A = [\{a, b\}, \{c\}, \{d, e, f\}]$.

i) List the elements of A. (2 Marks)

ii) Find $n(A)$. (2 Marks)

iii) Find the power set of A. (4 Marks)

QUESTION FIVE (20MKS)

a) Each student in Liberal Arts at some University has a mathematics requirement A and a science requirement B. A poll of 140 freshmen shows that: 60 completed A, 45 completed B, 20 completed both A and B. Use a Venn diagram to find the number of students who have completed:

i) At least one of A and B;

ii) exactly one of A or B;

iii) neither A nor B. (9 Marks)

b) Consider functions $f: A \to B$ and $g: B \to C$. Prove that if f and g are one-to-one, then the composition function $g \circ f$ is one-to-one. (4 Marks)

c) Draw a picture of the weighted graph G which is maintained in memory by the following vertex array DATA and weight matrix W: (7 Marks)

$$DATA: X, Y, S, T \; ; \; W = \begin{pmatrix} 0 & 0 & 3 & 0 \\ 5 & 0 & 1 & 7 \\ 2 & 0 & 0 & 4 \end{pmatrix}$$