SIU

(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

(MAIN EXAMINATION)

FOURTH YEAR SECOND SEMESTER EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN (MATHEMATICS)
(SMT & SME)
AND BACHELOR OF SCIENCE IN EDUCATION
(EDS, EDA & ETS)

COURSE CODE: MAT 426

COURSE TITLE: FOURIER SERIES

DATE: 20^{th} **April, 2023** TIME: **3 : 00 PM - 5 : 00 PM**

INSTRUCTIONS TO CANDIDATES:

- *Answer Question ONE (COMPULSORY) and ANY OTHER TWO questions.*
- *Do not write on the question paper.*

Time: 2 Hours

MMUST observes ZERO tolerance to examination cheating

This paper consists of 4 printed pages. Please turn over.

QUESTION ONE (COMPULSORY) [30 MARKS]

(a) Define Fourier Series as used in this context [2 marks]

(b) Sketch each of the following functions and state whether they are even, odd, or neither even nor odd [4 marks]

(i) $f(x) = \begin{cases} sin\ x & 0 \leq x < \pi \\ 0 & \pi < x \leq 2\pi \end{cases}$

(ii) $f(x) = \begin{cases} 3 & 0 < x < 5 \\ -3 & -5 < x < 0 \end{cases}$

(c) Find the Fourier series expansion of the triangular wave function defined by [4 marks]

$$f(t) = |t| \quad for \quad -1 \leq t \leq 1 \quad Period = 2$$

(d) Obtain the complex form of the Fourier series for $f(x) = 2$ on $0 \leq x \leq \pi$. [5 marks]

(e) Find the half range Cosine Fourier series expansion for [5 marks]

$$f(x) = x\pi - x^2 \quad on \quad 0 \leq x \leq \pi$$

(f) Prove that [5 marks]

$$\int_{-L}^{L} cos\frac{n\pi x}{L} cos\frac{m\pi x}{L} dx = \begin{cases} 0 & if & m \neq n \\ L & if & m = n \\ 2L & if & m = n = 0 \end{cases}$$

(g) Find the Fourier series expansion for [5 marks]

$$f(x) = 1 - \frac{2x}{\pi} \quad on \quad 0 \leq x \leq 2\pi \quad Period = 2\pi$$

QUESTION TWO [20 MARKS]

(a) State any two Dirichlets conditions for Fourier Series [2 marks]

(b) If $f(x) = \frac{[\pi - x]^2}{4}$, $0 \leq x \leq 2\pi$. Show that [7 marks]

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{cos\ nx}{n^2}$$

(c) Prove that [4 marks]

$$\int_{0}^{2\pi} cos\ nx\ dx = \int_{0}^{2\pi} sin\ mx\ dx = 0$$

(d) Find the Fourier series expansion for **[4 marks]**

$$f(x) = \begin{cases} -\pi & -\pi \leq x < 0 \\ x & 0 < x \leq \pi \end{cases}$$

and hence deduce that **[3 marks]**

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

QUESTION THREE **[20 MARKS]**

(a) Given the double Fourier cosine series **[4 marks]**

$$f(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} cos\, \frac{m\pi x}{L_1} cos\, \frac{n\pi y}{L_2}$$

Obtain its Fourier coefficient A_{mn}

(b) Find the Double Fourier series expansion of the following function **[4 marks]**

$$f(x,y) = x^2y^2, \qquad 0 < x < \pi,\ 0 < y < \pi$$

(c) Find the Fourier series expansion for **[5 marks]**

$$f(x) = x + x^2 \qquad -\pi \leq x \leq \pi$$

Hence, show that **[3 marks]**

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

(d) Convert the Complex Fourier form to its corresponding real form **[4 marks]**

$$(1-i)e^{ix} + 2e^{2ix} + (1+i)e^{-ix} + 2e^{-2ix}$$

QUESTION FOUR **[20 MARKS]**

(a) Prove that **[6 marks]**

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{cos\ nx}{n^2}, \qquad -\pi < x < \pi$$

Using Parseval's formula show that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

(b) Find the Fourier series representation of **[5 marks]**

$$f(x) = x^2 - 2 \qquad for \qquad -2 < x < 2$$

(c) Expand $f(x) = cos\ x$, $0 < x < \pi$, in Fourier sine series. **[4 marks]**

(d) Find the steady temperature in a bar rod whose end points are located at $x = 0$ and $x = 10$ if these end points are kept at 150^0C and 100^0C respectively. **[5 marks]**

QUESTION FIVE **[20 MARKS]**

(a) Let the function $f(x)$ defined on $[-\pi,\ \pi]$ i.e,(2π- periodic) be represented by Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n cos\ nx + b_n sin\ nx]$$

Calculate the coefficients a_0, a_n and b_n **[6 marks]**

(b) Prove that for $0 < x < \pi$ **[5 marks]**

$$x(\pi - x) = \frac{8}{\pi}[sin\ x + \frac{1}{3^3}sin\ 3x + \frac{1}{5^3}sin\ 5x + \cdots]$$

(c) Obtain the Fourier series representing the function **[5 marks]**

$$f(x) = |sin\ x| \quad for \quad -\pi < x < \pi$$

(d) Using pointwise convergence theorem show that $f(x)$ converges in $[-2, 2]$ **[4 marks]**

$$f(x) = \begin{cases} 0 & -2 \leq x < 0 \\ 2 - x & 0 < x \leq 2 \end{cases}$$