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(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY MAIN EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATIONS

FOR THE DEGREE
OF
BACHELOR OF SCIENCE IN GEOSPATIAL INFORMATION

COURSE CODE: MAT 126 COURSE TITLE: CALCULUS II

DATE: THURSDAY 20/04/2023   **TIME: 12.00 PM- 2.00PM**

INSTRUCTIONS TO CANDIDATES

Answer question **ONE** (COMPULSORY) and any other **TWO** questions

Time: 2 hours

QUESTION ONE (30 MARKS)

(a) Determine the line of asymptote for the curve $y = \dfrac{5x^2 + 8x - 3}{3x^2 + 2}$ (3 marks)

(b) (i) Find $\displaystyle\int_0^2 5e^{1-3x}\,dx$ (3 marks)

(ii) Evaluate $\displaystyle\int_0^{\ln 2} 4e^x \sinh x\,dx$ (3 marks)

(c) Use the mid-ordinate rule with three strips to estimate the area enclosed by the curve $y = \dfrac{1}{2}x^2 + 1$, $x = 0$, $x = 3$ and the $x - axis$ (4 marks)

(d) Determine the approximate value of $\displaystyle\int_0^2 x^4\,dx$ using Simpson's rule (5 marks)

(e) Find the area enclosed by the curve $y = x^2 - 2x + 2$ and the straight line $y = 5$ (6 marks)

(f) Sketch the curve $y = (x-1)(x+3)^2$ (6 Marks)

QUESTION TWO (20 MARKS)

(a) Find $\dfrac{dy}{dx}$ if $y^2 = x^2 + \sin xy$ (3 marks)

(b) Find $\displaystyle\int_0^{\pi/2} \cos^2 x \sin x\,dx$ (4 Marks)

(c) Find the slope of the circle $x^2 + y^2 = 25$ at (3,-4) (4 marks)

(d) Evaluate $\displaystyle\int_0^1 Sinh^2 x\,dx$ (4 marks)

(e) Use prismoidal rule to evaluate $\displaystyle\int_1^5 \left(3x - \dfrac{1}{3}x^2\right) dx$ (5 marks)

QUESTION THREE (20 MARKS)

(a) Decompose $\dfrac{7x+8}{2x^2 + 11x + 5}$ into partial fractions and hence evaluate $\displaystyle\int \dfrac{7x+8}{2x^2 + 11x + 5}\,dx$ (6 marks)

(b) Find the volume of revolution between the lines $x=2$ and $x=5$ and $y=2x^2$ about the x- axis (5 marks)

(c) Estimate the area bounded by the curve $y = \frac{1}{2}x^2 + 5$, the x-axis, the line x=1 and x=5 using trapezium rule. (5 marks)

(d) Determine $\int_{0}^{\frac{\pi}{2}} \sin^4 x\, dx$ (4 marks)

QUESTION FOUR (20 MARKS)

(a) Evaluate $\int \coth 5x\, dx$ (3 marks)

(b) Determine the line of asymptote for the curve $y = \frac{11x + 2}{2x^3 - 2}$ (4 marks)

(c) Find the tangent and normal to the curve $x^3 + y^3 - 9xy = 0$ at the point (2, 4) (4 marks)

(d) Find the length of the arc traced in the first quadrant by a curve whose equation is given by $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq 2\pi$ (4 Marks)

(e) Evaluate $\int_{0}^{2.5} \sqrt{16 - x^2}\, dx$ using trapezoidal rule, taking $n = 5$. (5 Marks)

QUESTION FIVE (20 MARKS)

(a) A particle moves so that its position vector is given by $r = \cos.\omega t\, i + \sin.\omega t\, j$ where ω is a constant. Show that:
  (i)   the velocity v of the particle is perpendicular to r (2 Marks)
  (ii)   the acceleration a is directed towards the origin and has magnitude proportional to the distance from the origin. (3 Marks)

  (iii) $r \times v$ = a constant vector (4 Marks)

(b) A particle with an initial velocity of $30\, m/s$ has a uniform acceleration of $9.8\, m/s^2$.
  (i)   Determine its velocity at time $t=3s$ (3 marks)
  (ii)   Find the displacement at time $t=6s$ (displacement at t=0 is 0) (3 Marks)

(c) The area bounded by the curves $x^2 = 4y$ and $y= 2x$ rotated round the y-axis. Determine the volume generated. (5 marks)