MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

(MMUST)

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER EXAMINATIONS

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAT 322**

COURSE TITLE: **OPERATIONS RESEARCH I**

DATE: TUESDAY 25 TH APRIL 2023 **TIME: 12 – 2.00PM**

INSTRUCTION TO CANDIDATES

Answer question **one** and any other **two** questions.

Question One (30 Marks)

a) A company is planning to determine its product mix out of three different products: P_1, P_2, and P_3. The monthly sales of the product P_1 is limited to a maximum of 500 units. For every two units of P_2 produced, there will be one unit of by-product which can be sold at the rate of KShs 20 per unit. The highest monthly demand for this by-product is 200 units. The contributions per unit of the products P_1, P_2 and P_3 are KShs 50, 70 and 60 respectively. The processing requirements of the products are as shown below.

Hours per unit

Process	P_1	P_2	P_3	Available hours
1	3	5	2	800
2	4	_	3	500
3	4	3	2	900

Formulate a Linear Programming (LP) model of the problem to find the optimum product mix to maximize the total contribution. (5 Marks)

b) Write the associated dual problem of the following primal problem.

$$\text{Maximize } Z = 17X_1+5X_2$$
$$\text{Subject to } 6X_1+8X_2 \leq 20$$
$$4X_1+2X_2 = 40$$
$$X_1, X_2 \geq 0$$

(6Marks)

Consider the following transportation model involving three sources and three destinations.
i)Develop a linear programming model for this transportation problem. (4 Marks)

		Destination			
		1	2	3	Supply
	1	20	10	15	200
Source	2	10	12	9	300
	3	25	30	18	500
Demand		200	400	400	1000

Determine the minimum transportation cost by using the northwest corner method. (8 Marks)

c) Solve the following LP problem using simplex method.

$$\text{Maximize } Z = 5X_1+10X_2+15X_3$$
$$\text{Subject to } X_1+2X_2+3X_3 \leq 12$$
$$X_1+3X_2+2X_3 \leq 10$$
$$X_1, X_2, X_3 \geq 0$$

(7 Marks)

Question Two (20 Marks)

a) Use dual simplex method and solve the following LP model

$$\text{Minimize } Z = 5X_1+6X_2$$
$$\text{Subject to } X_1+X_2 \geq 2$$

$4X_1+X_2 \geq 4$

$X_1, X_2 \geq 0$ (13 Marks)

b) Use graphical method to solve the LP given below.

Maximize $Z = 5X_1+4X_2$

Subject to $6X_1+ 4X_2 \leq 24$

$X_1+2X_2 \leq 6$

$-X_1+X_2 \leq 1$

$X_2 \leq 2$

$X_1, X_2 \geq 0$ (7 Marks)

Question three (20 Marks)

a) Explain clearly the two types of transportation problems. (4 Marks)

b) A company has three plants A, B, and C which supply to warehouses located at D, E, F,G and H. Monthly plant capacities are 700, 500 and 1000 units, respectively. Monthly warehouse requirements are 400, 400, 500, 400 and 500 units respectively. Unit transportation cost (in KShs) are given below. Determine an optimum distribution for the company in order to minimize the total transport cost. Formulate the transportation matrix and obtain the basic feasible solution using least cell method (10 Marks)

		Destination				
		D	E	F	G	H
Source	A	5	8	6	6	3
	B	4	7	7	6	5
	C	8	4	6	6	4

c) Use Kruskal's algorithm to find the minimum spanning tree for the table of arc distances given below.

Arc	Distance	Arc	Distance	Arc	Distance
1-2	4	2-3	6	3-6	9
1-3	2	2-4	6	4-5	4
1-5	3	3-4	5	4-6	8

(10 Marks)

Question Four (20 Marks)

a) Solve the LP using the result of its dual problem.

Maximize $Z = 24X_1+3X_2$

Subject to $2X_1+ 3X_2 \geq 10$

$4X_1+9X_2 \geq 15$

$6X_1+6X_2 \geq 20$

$X_1, X_2 \geq 0$ (15 Marks)

Question Five

a) Consider the details of a distance network shown below

Arc/Path	Distance	Arc/Path	Distance
1-2	8	3-6	12
1-3	5	4-6	12
1-4	7	5-8	17
1-5	16	6-7	13
2-3	9	6-8	9
2-6	3	6-9	25
2-7	4	7-9	26
3-4	7	8-9	6

i. Construct the distance network.

ii. Find the shortest path from node 1 to node 9 using Dijkstra's algorithm. (10 Marks)

b) A national market survey indicates that Baby Mikks manufacturers produce both baby milk and baby malt from two raw materials M1 and M2. The following table provides the basic data of the problem.

	Baby Milk	Baby Malt	Maximum daily availability (tons)
Raw material M1(tons/unit ton)	1	1	6
Raw Material M2 (tons/unit ton)	7	1	14
Cost per ton(millions of KSh)	2	3	

Baby Mikks wants to determine the optimum product mix of baby milk and baby malt that minimizes total daily cost of M1 and M2. Develop the linear programming model and use the Big M method to solve it. Interpret your results. (10 Marks)