720

(The University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER MAIN EXAMINATIONS

FOR THE DEGREE

IN

BACHELOR OF SCIENCE (SME/SMT/SST/EDA/EDS)

COURSE CODE: MAT 202

COURSE TITLE: LINEAR ALGEBRA II

DATE: 24/04/2023 **TIME:** 8.00-10.00am

INSTRUCTIONS TO CANDIDATES

- Section A is compulsory any other THREE questions from section B
- Do all the rough work in the answer booklet

TIME: 2 hours

Question One (30 Marks)

a) Verify that the following is an inner product in P_2

$$\langle p(x), q(x)\rangle = a_0b_0 + a_1b_1 + a_2b_2$$

where $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$. (4 marks)

b) Transform the set $S = \{1, x, x^2\}$ which is a basis for P_2 into an orthonormal basis using the Gram-Schimdt orthonormalization process with respect to the integral inner product on P_2 defined as

$$\langle p(x), q(x)\rangle = \int_{-1}^{1} p(x)q(x)dx$$ (5 marks)

c) Given $T:\mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation defined as $T(x, y) = (2x - 3y, x + y)$ and $B = \{(1, 2), (2, 3)\}$, $B' = \{(1, 3), (1, 4)\}$ are both bases for $\mathbb{R}^2$. Find

i) A i.e matrix of representation of T with respect to basis B.

ii) Transition matrix P from B' to B. Hence use P and A to obtain A' (i.e. matrix of representation of T with respect to B'). (7 marks)

d) Let $A = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$ use the eigen value method to derive an explicit formula for A^n and solve the system of differential equations $\frac{dx}{dt} = 2x - 3y$ and $\frac{dy}{dt} = 4x - 5y$ given that x =7 and y =13 when t = 0. (6 marks)

e) Identify the curve which is represented by the following quadratic equation by first putting it into standard conic form, $x^2 + 2xy + y^2 - x + y = 0$ (6 marks)

f) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 4 & -3 \\ 0 & 3 & 1 \\ 0 & 2 & -1 \end{pmatrix}$ by making use of the Cayley-Hamilton theorem. (3 marks)

Question Two (20 Marks)

a) Consider the vector space of polynomials with inner product defined by $\langle f(x), g(x)\rangle = \int_{-1}^{1} f(x)g(x)dx$ and the polynomials $f(x) = x$ and $g(x) = 1 + x$. Find

i) $\langle f(x), g(x)\rangle$

ii) $\|f\|$

iii) $\|g\|$

iv) Normalize f and g. (6 marks)

b) State and prove the Cauchy – Schwarz inequality. (7 marks)

c) If v_1, v_2, Λ, v_n are eigenvectors associated with distinct eigenvalues $\lambda_1, \lambda_2, \Lambda, \lambda_n$ of a matrix A_{nxn} then show that the set $\{v_1, v_2, \Lambda, v_n\}$ is linearly independent. (6 marks)

Question Three (20 Marks)

a) For the matrix $A = \begin{pmatrix} 2 & 0 & -2 \\ -1 & 2 & -1 \\ -2 & 0 & 2 \end{pmatrix}$

- i) write down the characteristic polynomial
- ii) write down the characteristic equation
- iii) find the eigen values and eigen vectors corresponding to each eigen value.
- iv) find the basis for each eigen space. (10 marks)

b) Find the minimal polynomial $m(\lambda)$ of the matrix

$$A = \begin{pmatrix} \alpha & \beta \\ 0 & \alpha \end{pmatrix}$$

(4 marks)

c) If λ_1 and λ_2 are eigen values of a symmetric matrix A associated with eigen vectors v_1 and v_2 respectively, show that v_1 and v_2 are orthogonal. (6 marks)

Question Four (20 Marks)

a) Determine if $B = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}$ is orthogonal matrix (4 marks)

b) Let V be the vector space of polynomials over $\mathbb{R}$ and define $\langle f(t), g(t) \rangle = \int_0^1 f(t)g(t)dt$.

Find the angle θ between u and v if $u = 2t - 1$ and $v = t^2$. (7 marks)

c) Show that a square matrix $A_{n \times n}$ is diagonalizable if and only if A has n linearly independent eigenvectors. (9 marks)

Question Five (20 Marks)

a) Consider the bases of $\mathbb{R}^3$; $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $B' = \{(1,1,1), (1,1,0), (1,0,0)\}$.

- i) Find the transition matrix P from B' to B.
- ii) Find the transition matrix Q from B to B'

iii) Verify that $Q = P^{-1}$

iv) Compute coordinate matrix of w with respect to B' where $w = (-5,8,5)$.(13 marks)

b) Let $T : V \to V$ be a linear transformation. Let A be matrix of T with respect to B and A' be matrix of T with respect to B'. *Show that* $A' = P^{-1}AP$ *where* P *is* transition matrix from B' *to* B. (7marks)