

160



**MASINDE MULIRO UNIVERSITY OF SCIENCE AND
TECHNOLOGY
(MMUST)**

**UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR**

SECOND YEAR SECOND SEMESTER EXAMINATIONS

**FOR THE DEGREE
OF
BACHELOR OF SCIENCE (MATHEMATICS WITH IT)
BACHELOR OF TECHNOLOGY (MATHEMATICS AND COMPUTER
SCIENCE)**

COURSE CODE: MAT 206
COURSE TITLE: ALGEBRAIC STRUCTURES

DATE: Tuesday, 18th April 2023 **TIME: 12 Noon - 2 pm**

INSTRUCTIONS TO CANDIDATES

Answer question ONE (COMPULSORY) and any other TWO questions

Time: 2 hours

a

This Paper Consists of 3 Printed Pages. Please Turn Over.

- (i) Show that $3 \times (9 \times 11) = (3 \times 9) \times 11$. (2 marks)
- (ii) Show that (M, \times) is a group. (5 marks)
- (iii) Show that this group is cyclic, and write down all possible generators of this group. (2 marks)

Question Four (20 marks)

- a. The set $G = \{1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20\}$ forms a group under multiplication modulo 21.
 - (i) Find the elements in the subgroup of (G, \times_{21}) generated by the element 5 and state its order.
 - (ii) Explain why (G, \times_{21}) has no subgroup of order 5. (5 marks)
- b. Let $(G, *)$ and (H, \circ) be isomorphic groups with identity elements e_G and e_H respectively, and let $f: G \rightarrow H$ be an isomorphism from G to H . Prove that $f(e_G) = e_H$ (5 marks)
- c. G and H are cyclic groups with $[G] = [H]$. Prove that $G \cong H$. (10 marks)

Question Five (20 marks)

Consider the set of rational numbers, $R = \mathbb{Q}$, with addition $x * y = x + y + 1$ and multiplication $x \circ y = x + y + xy$

- i. Show that $(R, *)$ is a group. Is the group commutative?
- ii. Show that $(R, *, \circ)$ is a ring.
- iii. Is R a commutative ring? Does it have an identity?