140

(The University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER MAIN EXAMINATIONS

FOR THE DEGREE

IN

BACHELOR OF SCIENCE (ECT AND ETM)

COURSE CODE: MAT 213

COURSE TITLE: LINEAR ALGEBRA

DATE: 12/04/2023 **TIME: 3.00-5.00 pm**

INSTRUCTIONS TO CANDIDATES

- Section A is compulsory any other THREE questions from section B
- Do all the rough work in the answer booklet

TIME: 2 hours

Question One (30 Marks)

a) Define the following terms
   i) A singular matrix **(2 Mark)**
   ii) A linear transformation **(2 Mark)**
   iii) A vector **(1 Mark)**
   iv) A spanning set of a vector space **(2 Marks)**

b) Let $u = (1, -2, 3)$ and $\boldsymbol{v} = (4, -2, -4)$. Find the dot product $\boldsymbol{u} \cdot \boldsymbol{v}$ and the unit vector $\widehat{\boldsymbol{v}}$ in the direction of $\boldsymbol{v}$. **(5 Marks)**

c) Suppose we know for a linear transformation T of $\mathbb{R}^2$ that $T\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$ and $T\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$. Find the matrix A so that $T(x) = Ax$. **(6 Marks)**

d) Use elementary row operations to produce the row echelon form of the following matrix. **(4 Marks)**

$$A = \begin{bmatrix} 2 & -2 & 4 & 0 \\ 0 & 2 & 8 & -7 \\ -3 & 4 & -2 & -5 \end{bmatrix}$$

e) Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ hence solve the linear system

$3x + y = 3 \;;\; 4x + 2y = -8$ **(4 Marks)**

f) Let $u = (2,5,3)$ and $\boldsymbol{v} = (1,6,4)$. Express the vector $\boldsymbol{w} = (7,7,3)$ as a linear combination of $\boldsymbol{u}$ and $\boldsymbol{v}$. **(4 Marks)**

Question Two (20 Marks)

a) Determine whether the three vectors $u = (1,2,3,2), v = (2,5,5,5)$ and $w = (2,6,4,6)$ are linearly independent or dependent. **(4 Marks)**

b) What is the vector projection of the vector $u = (2,3,1)$ onto $\boldsymbol{v} = (1, -2, 2)$ **(3 Marks)**

c) Find the basis vectors and the dimension for the kernel of a transformation represented by the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}.$$

(6 Marks)

d) Compute the inverse of the matrix $M = \begin{bmatrix} -4 & 0 & 5 \\ -3 & 3 & 5 \\ -1 & 2 & 2 \end{bmatrix}$. **(7 Marks)**

Question Three (20 Marks)

a) Let $v_1 = (2,5)$ and $v_2 = (1,3)$. Show that $\{v_1, v_2\}$ is a spanning set for $\mathbb{R}^2$. **(6 Marks)**

b) What is a rank of a matrix? **(2 Mark)**

c) Let A be an n square matrix. How would you compare the rank of A when it is invertible and when it is not invertible? If matrix A is not invertible, how do some rows of the matrix relate with one another and what does the row echelon form of the matrix look like? **(4 Marks)**

d) Find the rank of the matrix $A = \begin{pmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -1 & 3 \\ -1 & 4 & -3 & 14 \end{pmatrix}$ solve and classify the linear system

$x + y - 2z = 1;\ y - z = 3;\ \ -x + 4y - 3z = 14$ **(8 Marks)**

Question Four (13 Marks)

a). Find the angle and the distance between vectors $\boldsymbol{v} = (3, -1, 2)$ and $\boldsymbol{u} = (-1, 2, 3)$. **(8 Marks)**

b). Let V and W be vector spaces and $T: V \to W$ be a transformation. Prove that the image of T is a subspace of W. **(7 Marks)**

c). Define the term basis of a vector space hence, state the standard basis vector in the basis of $\mathbb{R}^3$. **(5 Marks)**

Question Five (13 Marks)

a) For the system $x - y + 3z = 1, y = -2x + 5, 9z - x - 5y + 7 = 0$
   i) Write the system in the matrix form $A\boldsymbol{u} = \boldsymbol{b}$ for $\boldsymbol{u} = (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$. **(4 Marks)**

   ii) Classify the system as either homogenous on non-homogeneous. Explain **(2 Marks)**

   iii) Write the augmented matrix for this system and find its row-reduced echelon form **(6 Marks)**

   iv) Find the solution of the system and identify the relationship between the planes represented by the system. **(3 Marks)**

b) Let the coordinates of a point A be $A(1, 0, -2)$ and $\boldsymbol{v} = (4, -2, 3)$ be a vector. Fine the vector P such that $P = OA + tv$ where $t = -2$. **(5 Marks)**