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(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (IN MATHEMATICS)

COURSE CODE: MAT 222
COURSE TITLE: ADVANCED CALCULUS

DATE: APRIL 19, 2023 TIME: 3.00-5.00PM

Instruction to the candidates:
Answer question ONE (COMPULSORY) and any other TWO questions
Time: 2 hours

This paper consists of 3 printed pages. Please turn over.

SECTION A: Answer ALL the questions in this section

QUESTION ONE - 30 MARKS (COMPULSORY)

(a) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+4y^2}.$$ [5 mks]

(b) Find the directional derivative of $f(x,y) = 4x - y^2e^{3xz}$ at the point $(3,-1,0)$ in the direction of the vector $v = -i + 4j + 2k$. [4 mks]

(c) Find the equation of the tangent plane and the normal to the surface $x^2y = 4ze^{x+y} - 35$ at the point $(3,-3,2)$ [5 mks]

(d) A cylindrical tank measuring 10ft tall has a diameter of 4ft. If the possible error involved in each of these measurements is $\pm 0.01ft$, find the percentage error in the volume of the tank using differentials. [5 mks]

(e) Find the Taylor series for $f(x) = cos4x$ about the point $x = 0$ [3 mks]

(f) Evaluate the iterated integral

$$\int_0^4 \int_2^6 (4x + 8y)dxdy$$

[3 mks]

(g) Determine if the given sequence converges or diverges. If it converges what is its limit?

$\{\frac{(-1)^{n-2}n^2}{4+n^3}\}_{n=1}^{\infty}$. [5 mks]

SECTION B: Answer any TWO questions from this section

QUESTION TWO - 20 MARKS

(a) A box, open at the top has a volume of 32ft. Find the dimensions that minimize the surface area of the box. [7 mks]

(b) Using Lagrange multipliers find the maximum and minimum values of the function $f(x,y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$ [7 mks]

(c) Determine if the following sequence is increasing, decreasing, not monotonic, bounded below, bounded above and/or bounded. [6 mks]

$\{\frac{2n^2-1}{n}\}_{n=2}^{\infty}$

QUESTION THREE - 20 MARKS

(a) Evaluate

$$\int\int\int_V 6z^2 dV$$

where V is the region below $4x + y + 2z = 10$ in the first octant. [7 mks]

(b) Evaluate the following integral by first converting to an integral in polar coordinates

$$\int_0^3 \int_{-\sqrt{9-x^2}}^0 e^{(x^2+y^2)} dy dx$$

[6 mks]

(c) Given the functions $z = x^{-2}y^6 - 4x$ and $x = u^2v$, $y = v - 3u$ use chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ [4 mks]

(d) Apply the implicit function theorem to find $\frac{dy}{dx}$ given $2y^3 + 4x^2 - y = x^6$ [3 mks]

QUESTION FOUR - 20 MARKS

(a) Find the Taylor series expansion for $f(x) = e^{-6x}$ about the point $x = -4$. [5 mks]

(b) The production level for a manufacturer is $f(x, y) = -4x + xy + 2y$. Assume that the total amount available for labour and capital is \$2000 and that units of labour and capital cost \$20 and \$4 respectively. Find the maximum production level in this case. [6 mks]

(c) Compute the Jacobian of the following transformation:$x(u, v) = 4u - 3v^2$ and $y(u, v) = u^2 - 6v$. [3 mks]

(d) Determine the relative extrema of $f(x, y) = -x^2 - 5y^2 - 10y + 8x - 13$ [6 mks]

QUESTION FIVE - 20 MARKS

(a) Apply the Alternating Series test to determine the convergence or divergence of

$$\sum_{n=1}^{\infty} \frac{n}{(-2)^{n-1}}$$

[5 mks]

(b) Apply the integral test to the series

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

[6 mks]

(c) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} 3(x-2)^n$$

[5 mks]

(d) Apply the Ratio test to

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

[4 mks]