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(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY MAIN EXAMINATIONS FOR 2022/2023 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF

BACHELOR OF SCIENCE IN GEOSPATIAL INFORMATION

COURSE CODE: MAT 223 COURSE TITLE: CALCULUS III

DATE: TUESDAY 25/04/2023 TIME: 12.00 PM- 2.00 PM

INSTRUCTIONS TO CANDIDATES

*Answer **Question ONE** and **any other TWO Questions***

Time: 2 hours

QUESTION ONE (30 MARKS)

a. (i) Show that matrices **A**= $\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ and **B**= $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ are not similar (3 marks)

(ii) Find the Eigen values of Matrix **B**= $\begin{pmatrix}-2 & -1\\ 5 & 2\end{pmatrix}$ (3 marks)

b. Find $\lim_{x\to\infty}\frac{2x+3}{5-x}$ (3 marks)

c. Evaluate $\iint_R \frac{x}{y}dxdy$ where R is the region $-1\le x\le 1, 1\le y\le 2$ (4 Marks)

d. Expand $f(x)=e^x$ as an infinite series up to x^4 (5 marks)

e. A thin plate covers a triangular region bounded by the X – axis and the lines $X=1$ and $y=2x$ in the first quadrant. The density of the plate at the point (x,y) is $d(x,y)=6x+6y+6$. Find: axis

(i) the mass of the plate (2 marks)
(ii) first moment about the X – axis (2 marks)
(iii) centre of mass about the coordinate axes (2 marks)

f. Given that $w=\ln(e^x+e^y+e^z)$, show that $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}=1$ (6 marks)

QUESTION TWO (20 MARKS)

a. Find $\int_0^\infty \tan\frac{dx}{1+x^2}dx$ (4 marks)

b. The area bounded by the curves $x^2=4y$ and the line $y=2x$ revolves round the y – axis. Determine the volume generated (5 marks)

c. Find the length of the circle of radius r defined parametrically by $x=r\cos t$ and $y=r\sin t, 0\le t\le 2\pi$ (5 marks)

d. A thin plate covers a triangular region bounded by the X – axis and the lines $X=1$ and $y=2x$ in the first quadrant. The density of the plate at the point (x,y) is $d(x,y)=6x+6y+6$. Find the three radii of gyration, R_x, R_y and R_0. (6 marks)

QUESTION THREE (20 MARKS)

a. Evaluate $\iint_R (x^2+y^2)\partial x\partial y$ where R is a rectangle $0\le x\le 2, 0\le y\le 3$ (5 marks)

b. The standard parameterization of the circle of radius 1 centred at the point $(0,1)$ in the y – axis is given by $x=\cos t, y=1+\sin t, 0\le t\le 2\pi$. Use this parameterization to find the area of the surface swept by revolving the circle about the X – axis (5 marks)

c. Given Matrix **A**= $\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$

i. State the characteristic polynomial of **A** (1 mark)

ii. State the characteristic equation (1 mark)

iii. Determine the dimension of the eigen space (2 marks)

iv. Determine the eigen vectors corresponding to each eigen value (3 marks)

v. Find the eigen space (3 marks)

QUESTION FOUR (20 MARKS)

a. Evaluate $\int_{0}^{\ln 2} 4e^{x} \sinh x dx \ \ x$ (3 marks)

b. Find the length of the asteroid $x = Cos^{3}t, y = Sin^{3}t, 0 \leq t \leq 2\pi$ (4 Marks)

c. Expand $f(x) = \sin x$ as an infinite series up to x^4 term. (4 marks)

d. Find the Taylor series for $x \sin x$ at $x = 0$ (4 marks)

e. Evaluate $\iint_R xy\partial x\partial y$ where R is the area under the quadrant of a circle $x^2 + y^2 = 4$ (5 marks)

QUESTION FIVE (20 MARKS)

a. Determine $\lim\limits_{x \to 2} \frac{x^3 + 2x^2 - 5x - 6}{x^2 - 4}$ (4 marks)

b. Verify Rolle's Theorem for the function $f(x) = x^2 + x - 6$ (5 marks)

c. Given that $f(x) = x^2$ is continuous in [0,2] and differentiable at some point c in (0,2), find the value of c such that $\frac{f(2) - f(0)}{2 - 0} = f^1(c)$ (5 marks)

d. Use Cramer's rule to determine the solution to the following system of equations

$$3x_1 + x_2 + 5x_3 = -2$$
$$-4x_1 + x_2 + 7x_3 = 10$$
$$2x_1 + 4x_2 - x_3 = 3$$

(6 marks)