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(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY MAIN EXAMINATIONS

2022/2023 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER EXAMINATIONS

FOR THE DEGREE
OF
BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAT 302 COURSE TITLE: REAL ANALYSIS II

DATE: TUESDAY 11/04/2023 TIME: 3.00 PM- 5.00 PM

INSTRUCTIONS TO CANDIDATES

Answer question **ONE** (COMPULSORY) and any other **TWO** questions

Time: 2 hours

QUESTION ONE (30 MARKS)

(a) (i) What is meant by the term critical point? (2 marks)

(ii) Determine the absolute extrema values of $g(t) = 8t - t^4$ on $[-2,1]$ (4 marks)

(b) Show that a constant function defined on a bounded closed interval is integrable (5 marks)

(c) Show that if a power series $\sum_{n=0}^{\infty} a_n x^n$ converges at some point $x_0 \in \mathbb{R}$ then it converges absolutely for any x satisfying $|x| < |x_0|$. (4 Marks)

(d) Use Implicit Function Theorem to find the derivative of $6x^4 - 7y^4 + 5z^2 = 2\sin(yz)$ (5 marks)

(e) Show that if $I \subset \mathbb{R}$ is an interval and $f : I \to \mathbb{R}$ is strictly monotone then the inverse $f^{-1} : f(1) \to I$ is continuous. (5 marks)

(f) Show that if P a partition of $[a,b]$ and P^* is any refinement of P ,then for a bounded function on $[a,b]$, $U(P^*, f) \leq U(P, f)$ for any partition P of $[a,b]$ (5 marks)

QUESTION TWO (20 MARKS)

(a) (i) Find the length of the circle of radius r defined parametrically by $x = r\cos t$, and $y = r\sin t$, $0 \leq t \leq 2\pi$. (4 mark)

(ii) Show that if $f_{(n)}$ is a sequence of functions defined on $A \subseteq \mathbb{R}$ that converges uniformly on A to a function f and each $f_{(n)}$ is continuous at $C \in A$ then f is continuous at C. (5 Marks)

(b) Show that if $g(x,y) = 1 - x^2 - 2y^2$ then $\lim_{(x,y)\to(x_0,y_0)} g(x,y) = 1 - x_0^2 - 2y_0^2$ (5 marks)

(c) Show that if f is a non-negative monotone decreasing integrable function on $[1,\infty]$ such that $f_{(n)} = x_n, n \in \mathbb{N}$ then $\sum_{n=1}^{\infty} x_n$ and $\int_1^{\infty} f(x)dx$ converge or diverge together. (6 Marks)

QUESTION THREE (20 MARKS)

(a) (i) State two properties of integrable functions (2 marks)

(ii) Find the length of the asteroid $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq 2t$ (7 marks)

(b) Show that if $T : X \to Y$ is a continuous bijective BC linear map from an $F - BC$ module X to an $F - BC$ module Y then T has a continuous BC linear inverse. (5 Marks)

(c) Find the absolute maximum and minimum values of $f(x) = x^2$ on $[-2,1]$ (4 marks)

(d) What is meant by the term total variation of the function f on [a, b]. (2 marks)

QUESTION FOUR (20 MARKS)

(a) Suppose $f_n \to f$ point wise on the closed interval $[a,b]$ and assume that each f_n is differentiable. Show that if $(f^1_{(n)})$ converges uniformly on $[a,b]$ to a function g, then the function f is differentiable and $f^1 = g.$ (10 Marks)

(b) Show that the function $f:[0,a] \to \mathbb{R}, a > 0$ defined by $f(x) = x^2 \in [0,a]$ is integrable on $[a,b]$ (10 Marks)

QUESTION FIVE (20 MARKS)

(a) (i) What is meant by the term Riemann - Stieltjes integral of f on $[a,b]$ (2 Marks)

(ii) Show that the function $f:[0,1] \to \mathbb{R}$ defined by $f(x) = \begin{cases} 1, x \in Q \\ 0, x \notin \mathbb{Q} \end{cases}$ is not integrable over $[0,1]$ (5 marks)

(b) Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2,3]$ (5 marks)

(c) Show that a monotone sequence is convergent if and only if it is bounded (8 Marks)