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(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
(MAIN)

THIRD YEAR SECOND SEMESTER EXAMINATION

FOR THE

BACHELOR OF SCIENCE ENGINEERING (ECE, CSE, & MIE)

COURSE CODE: **MAT 302/310**

COURSE TITLE: **DIFFERENTIAL EQUATIONS**

DATE: 11TH APRIL, 2023 **TIME: 3.00-5.00 PM**

A. INSTRUCTIONS TO CANDIDATES

- Answer question **ONE** and **ANY OTHER TWO** questions.

Time 2 Hours

MMUST observes ZERO tolerance to examination cheating

QUESTION ONE (30 MARKS) COMPULSORY

a) Differentiate between Partial Differential Equation and Ordinary Differential Equation. (2 Marks)

b) Solve the differential equation $\sqrt{y}dx+(1+x)dy=0 \quad y(1)=1$. (4 Marks)

c) According to Newton's law of cooling the rate at which a substance cools in moving air is proportional to the difference between the temperature of a substance and that of the air. If the temperature of the air is 30^0 and the substance cools from 100^0 to 70^0 in 15 minutes, find when the temperature will be 40^0. (5 Marks)

d) Solve the ordinary differential equation $y''-3y'-4y=2\sin x$. (5 Marks)

e) Solve the Bernoulli equation $\frac{dy}{dx}+\frac{1}{x}y=xy^2$. (4 Marks)

f) A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form $y=a\sin\frac{\pi x}{L}$ from which it's released at a time $t=0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t)=a\sin(\frac{\pi x}{L})\cos\frac{\pi ct}{L}$. (5 Marks)

g) Find the general solution of the equation $(x-y)p+(y-x-z)q=z$ and passing through the circle $z=1,\ x^2+y^2=1$. (5 Marks)

QUESTION TWO (20 MARKS)

a) Classify the following differential equation. (2 Marks)

i. $\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=z$

ii. $(\frac{d^2y}{dx^2})^3+\frac{d^2y}{dz^2}+y=kxy$

b) Solve the equation $\frac{dy}{dx}=\frac{y}{x}+x\sin\frac{y}{x}$. (5 Marks)

c) Find the family of trajectories of the family of curves $xy=c$. (5 Marks)

d) Using the method of variation of parameters, solve the differential $y'' + y - \sec x$

(5 Marks)

e) If the population of a country doubles in 50 years, in how many years will it treble under the assumptions that the rate of increase is proportional to the number of inhabitants?

(3 Marks)

QUESTION THREE (20 MARKS)

a) Form a partial differential equation from $z = f(x^2 - y^2)$. (4 Marks)

b) Using Charpits method, solve $(p^2 + q^2)y = qz$. (5 Marks)

c) Using the method of separation of variables solve

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$. (6 Marks)

d) Find the general solution of the partial differential equation $2p + 3q = 1$ using Lagrange's method.

(5 Marks)

QUESTION FOUR (20 MARKS)

a) Differentiate between the following terms. (4 Marks)

i. Homogeneous and non-homogeneous differential equation

ii. Linear and non-linear differential equation.

b) Eliminate the arbitrary function form $z = f(x + iy) + g(x - iy)$. (5 Marks)

c) Find a differential equation associated with the solution $y = Ae^{2x} + Be^{x} + C$.

(5 Marks)

d) Determine whether the following differential equation is homogeneous $(y^2 + 2xy)dx - x^2dy = 0$ and hence solve it.

(6 Marks)

QUESTION FIVE (20 MARKS)

a) Solve the differential equation $y'' - 4y' - 5y = 0$ $y(0) = 1$ $y'(0) = 2$. (5 Marks)

b) Determine whether the following differential equation is exact and hence solve it

$3y + e^t + (3t + \cos y)\frac{dy}{dt} = 0$. (5 Marks)

c) Find the equation of the integral surface $2y(z-3)p+(2x-z)q=y(2x-3)$ which passes through the circle $z=0$ $x^2+y^2=2x$. (5 Marks)

d) If takes two years for 3g of radio-isotope to decay to 0.9g. Determine the half-life T of the isotope and the time it will take amount to 0.4g. (5 Marks)