625

(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)
(MAIN EXAMINATIONS)

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER EXAMINATIONS

FOR THE DEGREE OF

BACHELOR OF SCIENCE (SMT, SME, ETS, SPA, ETC, ETM, ETE) & BACHELOR OF EDUCATION

COURSE CODE: **MAT 304**

COURSE TITLE: **COMPLEX ANALYSIS I**

DATE: 19TH APRIL, 2023 **TIME: 3.00 – 5.00 PM**

Instructions to candidates:
Answers question ONE (COMPULSORY) and any other TWO questions.
Time: 2 hours
This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Let $z = 1 - i$. Find z^{10}. **(5 Marks)**

b) Find the Taylor series of Show that $f(z) = cosz$ about $z = 0$ **(3 Marks)**

c) If $z = 4 + 3i$, determine z^3 in polar and Cartesian **(4 Marks)**

d) Determine the pole(s) and the residue(s) for $f(z) = \frac{1}{(1+z^2)(z+2)}$ **(5 Marks)**

e) Using the Cauchy-Riemann equations, show that $f(z) = z^3$ is analytic in the entire $z - plane$ **(3 Marks)**

f) Express $\frac{1+2i}{1-3i}$ in the form $r(\cos\theta + i\sin\theta)$ **(5 Marks)**

g) Prove that $u = e^x(xsin\, y - y\cos y)$ is harmonic **(5 Marks)**

QUESTION TWO (20MARKS)

a) Prove Cauchy-Riemann equations for the function $f(z) = z^2 - 5z + 1$ **(7 Marks)**

b) The real part of a complex function $f(z)$ is $u(x, y) = x^3 - 3xy^2$, find

  i. $v(x, y)$ so that $f(x, y) = u + iv$ is analytic

  ii. Express $f(x, y)$ in terms of z **(7 Marks)**

c) Show that the function $w = e^z$ is analytic in the entire complex plane. **(6 Marks)**

QUESTION THREE (20MARKS)

a) Evaluate $\int_0^{1+i}(x - y + ix^2)dz$ **(10 Marks)**

  i. Along the straight line from $z = 0$ to $z = 1 + i$

  ii. Along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$

  iii. Along the imaginary axis from $z = 0$ to $z = i$ and then along a line parallel to the real axis from $z = i$ to $z = 1 + i$.

b) Evaluate $\int_c \frac{z+1}{z^3-4z}dz$ where c is **(10 Marks)**

  i. $|z + 2| = \frac{3}{2}$

  ii. $|z - 2| = \frac{3}{2}$

QUESTION FOUR (20MARKS)

a) Find the residues of $f(z) = \frac{z^2+2z}{(z+1)^2(z^2+4)}$ at all its poles and hence evaluate $\oint_c f(z)dz$ **(10 Marks)**

b) Find the first four terms of a Tylor's series expansion of the function $f(z) = \frac{1}{(z-1)(z-3)}$ **(10 Marks)**

QUESTION FIVE (20MARKS)

a) If $z_1 = r_1(\cos\theta_1 + i\,sin\,\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\,sin\,\theta_2)$ prove $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos\theta_1 - \theta_2) + i\,sin\,(\theta_1 - \theta_2)$ **(4 Marks)**

b) Using the definition of limits, show that:-

i) $\lim_{z\to i}(7z-1) = 7i-1$ **(3 Marks)**

ii) $\lim_{z\to 1}(2z+1) = 3$ **(3 Marks)**

c) Find the image of the right half plane X≥ 0 under the mapping $W = \frac{z-1}{z+1}$ **(5 Marks)**

d) Find the Laurent series expansion for $f(z) = \frac{1}{z(z+1)^3}$ about the pole z=−2 **(5 Marks)**