(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATION

2022/2023 ACADEMIC YEAR

(MAIN EXAMINATIONS)

FIRST YEAR SECOND SEMESTER EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ENGINEERING
(MIE, ECE, CSE & BTB)

COURSE CODE: MAT 103/162

COURSE TITLE: PURE MATHEMATICS II

DATE: 24th April, 2023   TIME: 8: 00 AM - 10: 00 AM

INSTRUCTIONS TO CANDIDATES:

- *Answer Question ONE (COMPULSORY) and ANY OTHER TWO questions.*
- *Do not write on the question paper.*

Time: 2 Hours

MMUST observes ZERO tolerance to examination cheating

This paper consists of 3 printed pages. Please turn over.

QUESTION ONE (COMPULSORY) [30 MARKS]

(a) Using matrix method solve the following simultaneous equations [4 marks]

$$3x + 5y = 7$$
$$4x - 3y = 19$$

(b) Given two matrices $\mathbf{A}$ and $\mathbf{B}$, simplify $(A - B)^2 + (2B - A)^2 + 3BA$ [3 marks]

(c) Evaluate $\int \cos(3x + 7)\, dx$ [3 marks]

(d) A particle moves in a straight line so that after t seconds its acceleration is given by $a(t) = (6t - 12)\ m/s^2$. If its position at time $t = 0$ is $2\ m$ and the velocity is $9\ m/s$;

    (i) Determine velocity at any time t. [3 marks]

    (ii) Find the displacement at any time t. [2 marks]

(e) Evaluate $(2 + i)^3 + (2 - i)^2$ [3 marks]

(f) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 7 & 11 \end{bmatrix}$ [3 marks]

(g) Evaluate the integral by partial fractions [5 marks]

$$\int \frac{5}{x^2 - x - 6}\, dx$$

(h) Find the value of x such that the determinant of $|\mathbf{A}| = 12$, if $A = \begin{bmatrix} x^2 & 2 \\ 6 + x & 2 \end{bmatrix}$ [4 marks]

QUESTION TWO [20 MARKS]

(a) Determine the equation of the tangent line to the graph of $x^2 - 3xy + 2y^2 - 2x - 4 = 0$ at point $(1, -1)$ [4 marks]

(b) Divide $3 + 4i$ by $7 - 3i$ [3 marks]

(c) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ Verify that

    (i) $(AB)C = A(BC)$ [4 marks]

    (ii) $A(B + C) = AB + AC$ [4 marks]

(d) Find x, y if $(2 + i)^3 + (3 - i)(x - iy) = x + iy$ [4 marks]

(e) Prove that $|B| = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = 1$ [1 mark]

QUESTION THREE [20 MARKS]

(a) Find $(1 - i\sqrt{3})^5$ using De Moivre's theorem [4 marks]

(b) Given the matrix $A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$ Find

    (i) The determinant of A. [2 marks]

    (ii) Matrices of minors and Co-factors of A. [5 marks]

    (iii) Adjoint of A and inverse of A. [2 marks]

(c) Convert $1 + i$ into polar form [3 marks]

(d) Find the derivative of $y = (x^2 + 1)^3 (x^3 + 3)^2$ using appropriate method [4 marks]

QUESTION FOUR [20 MARKS]

(a) Solve the following system of equations using Cramer's rule [7 marks]

$$5x - 7y + z = 11$$
$$6x - 8y - z = 15$$
$$3x + 2y - 6z = 7$$

(b) Evaluate $\int \sin^5 \theta \, d\theta$ [4 marks]

(c) Differentiate the functions

    (i) $y = 3\sin 4x - \cos^2 x$ [3 marks]

    (ii) $y = 3xe^{5x} + \sin x^2$ [3 marks]

    (iii) $y = \ln(3x^2 + 2x) + e^{x^2}$ [3 marks]

QUESTION FIVE [20 MARKS]

(a) Evaluate the derivative [3 marks]

$$f(x) = \frac{x^2 + 2}{x^3 - 3}$$

(b) Evaluate the definite integral [3 marks]

$$\int_{-2}^{3} (4 - x^2) \, dx$$

(c) Find the six roots of $z = -8$ and graph these roots in the complex plane. [5 marks]

(d) Given the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Show that $A^2 - 4A - 5I = 0$, where I is a unit matrix and 0 in a null matrix of order 3 respectively. [6 marks]

(e) Find the area bounded by the curve $y = 4e^{2x}$ between $x = -1$ and $x = \frac{1}{2}$ [3 marks]

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