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(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

(MMUST)

UNIVERSITY MAIN EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATIONS

FOR THE DEGREE
OF
MASTER OF SCIENCE IN PURE MATHEMATICS

COURSE CODE: MAT 802 COURSE TITLE: GENERAL TOPOLOGY II

DATE: THURSDAY 27/04/2023 **TIME: 8.00 AM-11.00 PM**

INSTRUCTIONS TO CANDIDATES

Answer any other **THREE** questions

Time: **3** hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (20 MARKS)

(a) Demonstrate that a regular T_1 −space whose topology has a $\sigma-$ locally finite base is metrizable. (7 marks)

(b) Show that every sequence in a topological space has a cluster point if and only if every infinite set has $\omega-$ accumulation points. (6 marks)

(c) Illustrate that the product of a Tychonoff space is a Tychonoff space. (7 marks)

QUESTION TWO (20 MARKS)

(a) Show that a regular space whose topology has is a $\sigma-$ locally finite base is normal. (5 marks)

(b) Demonstrate that for a family F of functions on a set X to a topological space Y to be compact relative to the topology of point wise convergence it is sufficient that:

(i) F be point wise closed in Y^X

(ii) For each point x of X the set $F[x]$ has a compact closure. (5 marks)

(c) Show that the compact open topology τ_o contains the topology τ of point wise convergence. Moreover the space $(F.,\tau_o)$ is a Hausdorff space if the range space Y is Hausdorff, and is regular if Y is regular and members of F are continuous. (10 marks)

QUESTION THREE (20 MARKS)

(a) Show that each regular Lindelof space is normal. (4 marks)

(b) Show that a topological space X is compact if and only if each net in X has a cluster point. (8 marks)

(c) Given that K is a subset of $\mathbb{R}$ show that the following statements are equivalent:

(i) K is compact

(ii) K is closed and bounded

(iii) Any open cover for K has a finite sub cover. (8 marks)

QUESTION FOUR (20 MARKS)

(a) (i) Show that the closure of a connected set is connected. (3 marks)

(ii) Show that if F is a family of connected subsets of a topological space and no two members of F are separated, then $\bigcup\{A : A \in F\}$ is connected. (3 marks)

(b) Show that if X is a T_1 $-$space then the following are equivalent:

(i) X is regular and there is a countable base for its topology
(ii) X is homeomorphic to a subspace of the cube Q^{ω}
(iii) X is metrizable and hence separable. (6 marks)

(c) State and prove Urysohn's Lemma (8 marks)

QUESTION FIVE (20 MARKS)

(a) Show that for a topology space to be a Tychenoff space it is necessary and sufficient that it be homeomorphic to a subspace of a cube. (5 marks)

(b) Show that a topological space is compact if and only if each family of closed sets which has the finite intersection property has a non-void intersection. (6 marks)

(c) If $P: X \rightarrow Y$ is a closed continuous surjective map such that $P^{-1}(y)$ is compact for each $y \in Y$, show that if Y is compact, then X is compact. (9 marks)