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(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATIONS

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: STA 808

COURSE TITLE: TIME SERIES ANALYSIS AND FORECASTING

DATE: 27-04-2023 **TIME: 2.00 - 5.00PM**

INSTRUCTIONS TO CANDIDATES

Answer any **THREE** questions.

TIME: 3 Hours

MMUST observes ZERO tolerance to examination cheating

QUESTION ONE (20 MARKS)

(a). (i). Define the spectral density function of a time series (4 marks)

(ii). Consider the AR(1) time series. Show that its spectral density function given by

$$f(w) = \frac{\sigma^2}{\pi(1-2\emptyset_1 \cos \omega + \emptyset_1^2)}$$ (8 marks)

where w and ϕ_1 are some constants

(b). Consider the process $Y_t = \varepsilon_t + 2\,\varepsilon_{t-1} + 0.8\,Y_{t-1}$ where $\{\varepsilon_t\}$ is sequence of independent random variables with mean 0 and variance σ^2. Determine the

auto-correlation function ρ_k $k = 0, 1, 2, 3,...$ (8 marks)

QUESTION TWO (20 MARKS)

(a). Consider the AR(2) process given by $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + e_t$ where α_1 and α_2 are some real constants and $\{e_t\}$ is a random variable with mean zero and variance σ^2. Find the condition for the process to be stationary.

(6 marks)

(b). Suppose $\sqrt{\alpha_1^2 + 4\alpha_2 < 0}$, show that the auto-correlation coefficient solution of X_t is given by

$$\rho(h) = (-\alpha_2)^{\frac{h}{2}} \frac{\sin(\theta h + \varphi)}{\sin\varphi}$$ (14 marks)

where $\cos\theta = \frac{\alpha_1}{2(-\alpha_2)}$ and $\cot\varphi = \frac{1+\alpha_2}{1-\alpha_2}$

QUESTION THREE (20 MARKS)

(a). Consider the MA(1) process X_t given by $X_t = e_t + \theta e_{t-1}$ where $\{e_t\}$ is the white noise process with zero mean and variance, σ^2. Show that the corresponding normalized spectral density function

$$f^*(\lambda) = \frac{1}{2\pi}\left\{1 + \frac{2\theta}{1+\theta^2} Cos\,\lambda\right\}$$ (8 marks)

(b). Consider the AR (2) process

$$X_t = X_{t-1} - 0.5X_{t-2} + e_t$$

(i). Determine whether the process is stationary

(ii). Determine the auto correlation function for the process. (12marks)

QUESTION FOUR (20 MARKS)

(a). Let $X_t = e_t \cos \lambda t + e_{t-1} \sin\lambda t$ where $\{e_t\}$ is white noise, λ is some constant. Show (X_t) is stationary. (4 marks)

(b). Consider the AR MA(2, 2) process given by

$$X_t - X_{t-1} - \propto X_{t-2} = e_{t-2} + e_{t-1} - 6\, e_t$$ where $\propto$ is some constant

and $\{e_t\}$ is a sequence of white noise. Show that the process is invertible.

(6 marks)

(c). Consider the process (X_t) given by $X_t = X_{t-1} + e_t$

where $\{e_t\}$ is the white noise process with mean zero and variance σ^2.

(i). Find the mean and variance of the process

(ii). Hence, or otherwise show that it is non-stationary (iii). Show that the process

$$Y_t = \Delta X_t$$ is stationary.

(iv). Determine the auto correlation function, and hence, the correlogram of (Y_t). (10 marks)

QUESTION FIVE (20 MARKS)

(a). Describe the main stages in setting up the Box-Jenkens forecasting model.

(8 marks)

(b). For the model $(1-B)(1-0.2B)X_t = (1 - 0.5B)e_t$ where $\{e_t\}$ is a sequence of white noise with mean zero and variance σ^2, and B is the backward shift operator, find forecast for one and two stages ahead and recursive expressions for 3 or more steps. (12 marks)