(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE

OF

MASTER OF SCIENCE (APPLIED MATHEMATICS)

COURSE CODE: MAT 851

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: 20th April ,2023 **TIME:** 2pm -5pm

INSTRUCTIONS TO CANDIDATES

- Answer question ONE (COMPULSORY) and any other TWO questions

Time: 3 hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (20mks)

a) Explain the different between sink, source and saddle equilibrium points. (3mks)

b) Expand $f(x) = sinx, 0 < x < \pi$ in a Fourier cosine series. (4mks)

c) Solve $y''(t) + y(t) = 1$ given $y(0) = 1, y'(0) = 0$ by use of Laplace transforms (4mks)

d) Discuss the existence of solutions, stating their nature for the following ODE's

i) $y' = 2x \quad y(0) = 1$

ii) $xy' = y - 1 \quad y(0) = 1$

iii) $y'^2 + y^2 + 1 = 0 \quad y(0) = 1$ (3mks)

e) Show that the origin is locally stable for a mathematical pendulum $\dot{x}_1 = x_2, \dot{x}_2 = -\frac{g}{l}\sin x_1$. Use a Lyapunov function candidate $v(x) = (1 - \cos x_1)gl + \frac{l^2 x^2}{2}$ (3mks)

f) Use elimination method to solve $\begin{matrix} \dot{x} = x + 3y \\ \dot{y} = x - y \end{matrix}$ (3mks)

QUESTION TWO (20mks)

a) Find the quadratic Lyapunov function for the system $\dot{x} = Ax = \begin{bmatrix} -1 & 4 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Is the system stable or asymptotically stable? (4mks)

b) Find a Fourier series for $f(x) = x^2, 0 < x < 2,$ hence evaluate $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ (4mks)

c) Prove that $e^{At}e^{Bt} = e^{(A+B)t}$ $\forall$t iff $BA = AB$ (2mks)

d) Solve the difference equation $y_{k+1} = 2y_k \ ; k = 0,1,2 \ldots ; y_o = 3$ (3mks)

e) Let γ^+ be a positive semi orbit in a closed bounded subset K of R^2 and suppose K has only a finite number of critical points, then proof that the following is satisfied (7mks)

i) $\omega(\gamma^+)$ is a critical point

ii) $\omega(\gamma^+)$ is a periodic orbit

iii) $\omega(\gamma^+)$ contains a finite number of critical points and a set of orbits γ_i with $\alpha(\gamma_i)$ and $\omega(\gamma^+)$ consisting of a critical point for each orbit γ_i

QUESTION THREE (20mks)

a) Solve the system of the differential equation $\frac{dx}{dt} = x + 3x$, $\frac{dy}{dt} = 5x + 3y$ (6mks)

b) Show that the $\alpha-$ *and* $\omega-$ limit sets of an orbit γ are closed and invariant. Furthermore, if $\gamma^{+}(\gamma^{-})$ is bounded, then the $\omega-(\alpha-)$ limit set is non empty, compact and connected, $dist(\phi(t,p),\omega(\gamma(p))\to 0$ as $t\to\infty$ and $dist(\phi(t,p),\alpha(\gamma(p))\to 0$ as $t\to\infty$. (8mks)

c) Expand $f(x) = x^2, 0 < x < 2\pi$ in a Fourier series if the period is 2π (6mks)

QUESTION FOUR (20mks)

a) Discuss the existence and unique solution for the IVP $y' = \frac{2y}{x}, y(x_0) = y_0$ (5mks)

b) Find the general solution to the system (6mks)

$$\frac{dx}{dt} = -4x + y + z$$
$$\frac{dy}{dt} = x + 5y - z$$
$$\frac{dz}{dt} = y - 3z$$

c) Classify all of the equilibrium points of the nonlinear system $\dot{x} = f(x)$ with $f(x) = \begin{bmatrix} x_1^2 - x_2^2 - 1 \\ 2x_2 \end{bmatrix}$ (7mks)

d) Find $L(te^{3t})$ (2mks)

QUESTION FIVE (20mks)

a) Prove that the set W is open, g is continuous and increasing on W and the sequence $\{g^k(w)\}\ k = 0,1,\ldots n \le \infty$ is monotonic where $g^k(w) = g(g^{k-1}(w),\ k = 1,2,\ldots g^0(w) = w$ (9mks)

b) Show that if $x(t)$ is an n x n matrix solution of $\dot{x} = A(t)x$, then either $\det x(t) \ne 0$ for all t or $\det x(t) = 0$ for t (5mks)

c) Discuss the stability of the nonlinear system (6mks)

a)
$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_2 + x_1^2$$
$$\dot{x}_3 = x_3 + x_1^2$$

b)
$$\dot{x}_1 = -x_2^3$$
$$\dot{x}_2 = x_1^3$$