# MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

# (MMUST)

## UNIVERSITY EXAMINATIONS

## 2022/2023 ACADEMIC YEAR

## FIRST YEAR SEMESTER EXAMINATIONS

## FOR THE DEGREE OF

## MASTERS IN APPLIED MATHEMATICS

**COURSE CODE:** **MAT 853**

**COURSE TITLE:** **PDE I**

**DATE: WEDNESDAY 19/4/2023** **TIME: 3.00PM – 5.00PM**

**INSTRUCTION TO CANDIDATES**

Answer any three questions.

**Question One (20 Marks)**

a) The temperature T(x,t) in a stationary medium $x \geq 0$ is governed by the heat conduction equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ making the change of the variable (x, t) to (u, t) ,where $u = \frac{x}{2\sqrt{t}}$.

Show that $4t\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial u^2} + 2u\frac{\partial T}{\partial u}$ (5 Marks).

b) Reduce the partial differential equation $u_{xx} + 5u_{xy} + 6u_{yy} = 0$ to canonical form and find the general equation. (5 Marks)

c) Find the three general solutions and the particular solution of the Laplace's equation $U_{xx} + U_{yy} = 0$ given the boundary conditions U(0,y) = U(L,y) = U(x,0) = 0, and also that U(x,a) = $\sin\frac{n\pi x}{L}$. (10 Marks)

**Question Two (20 Marks)**

d) Given that the standard form of the partial differential equation $4u_{xx} + 8u_{xy} + u_{yy} = 0$ is $u_{\phi\theta} = 0$, using the method of classification of second order differential equations classify it as parabolic or hyperbolic or elliptic. Showing all the work solve the given PDE then transform it to the standard form.

**Question Three (20 Marks)**

a) Find the characteristics of the pde $xu_{xx} + (x-y)u_{xy} - yu_{yy} = 0$ $x > 0$, $y > 0$ then show that it can be transformed into canonical form $(\xi^2 + 4\eta)u_{\xi\eta} + \xi u_\eta = 0$, where $\xi$ and $\eta$ are suitable chosen canonical coordinates. Use this to obtain the general form

$$u(\xi,\eta) = f(\xi) + \int^{\eta} \frac{g(\eta')d\eta'}{(\xi^2 + 4\eta')^{\frac{1}{2}}}$$ (10 Marks)

b) Solve $F_t = a^2 F_{xx}$ (10 Marks)

**Question Four (20 Marks)**

a) Show that the one dimensional wave equation $u_{tt} - c^2 u_{xx} = 0$ can be transformed to $u_{\mu\eta} = 0$ and the general solution given as $u = F(\mu) + G(\eta)$ where μ and η are functions of *x* and *t*. (10 Marks)

b) V is the potential and I is the current at time t at a point P of the cable at a distance x from a given point, and R, L, C and G are the resistance, inductance, capacitance and leakage to the ground per unit length of the cable respectively, each assumed to be constant. Derive the general transmission line equations and in particular show that the telegraph equations are $\frac{\partial^2 V}{\partial x^2} = RC\frac{\partial v}{\partial t}$ and $\frac{\partial^2 I}{\partial x^2} = RC\frac{\partial I}{\partial t}$ and the radio equations are $\frac{\partial^2 V}{\partial x^2} = CL\frac{\partial^2 v}{\partial t^2}$ and $\frac{\partial^2 I}{\partial x^2} = CL\frac{\partial^2 I}{\partial t^2}$ (10 Marks)

c)

**Question Five (20 Marks**

a) Determine the characteristic curve of the equation $u_{xx} + 4u_{xy} + u_{yy} = 0$ (4 Marks)

b) By separation of variables solve $4\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y}$ (6 Marks)

c) Find $u(x,t)$ in the form $u(x,t) = \frac{a_0 t}{2} + \sum_{n=1}^{\infty} a_n(t)\cos\frac{n\pi x}{L} + b_n(t)\sin\frac{n\pi x}{L}$ if

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 \\ u(x,0) = g(x), u_t(x,0) = h(x), 0 \angle x \angle L \\ u(0,t) = 0, u(L,t) = 0, t \geq 0 \end{cases}$$

(10 Marks)