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(*University of Choice*)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATIONS
MAIN EXAM

FOR THE DEGREE OF
MASTER OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: **MAT 856**

COURSE TITLE: **APPLIED DYNAMICAL SYSTEMS II**

DATE: 21st April, 2023 **TIME: 8.00-11.00 AM**

INSTRUCTIONS TO CANDIDATES

- Answer question ONE (COMPULSORY) and any other TWO questions

QUESTION ONE (30 MARKS)

a) Distinguish between α limit set and ω limit set (3 Marks)

b) Compute the invariant stable and unstable manifolds $W^{S}_{loc}(0,0)$ and $W^{u}_{loc}(0,0)$ respectively of the unforced doffing Van der Pol oscillator (7 Marks)

$$x' = y$$
$$y' = x - x^3 - \delta y$$

c) Compute the Melkinov's function $M(\mu,\alpha)$ for the Lienard equation (5 Marks)

$$x' = y - \in [\mu_1 x + \mu_2 x^2 + \mu_3 x^3]$$
$$y' = -x$$

d) Compute the generalized Lyapunov type numbers γ and σ for the system $X' = AX$ where

$$A = \begin{pmatrix} \lambda & 0 \\ 0 & -\mu \end{pmatrix} \lambda, \mu > 0$$ (8 Marks)

e) Use polar coordinates to analyze the system below and show that there exist an attracting limit cycle (7 Marks)

$$x' = x - y - x(x^2 + y^2)$$
$$y' = x + y - y(x^2 + y^2)$$

QUESTION TWO (15 MARKS)

a) Define the shift map in symbolic dynamics of a horse shoe map as $\sigma(a)_i = a_{i+1}$. Show that (7 Marks)

(i) $\sigma^2\{\overline{10.10}\} = \{\overline{10.10}\}$

(ii) The period of orbit $\{\overline{001.001}\}$ is three

(iii) $\{\overline{00.00}\} = \{\overline{11.11}\}$ are fixed points

b) Show that the system

$$x' = y$$
$$y' = -1 + x^2$$

Has a homoclinic orbit connecting a hyperbolic fixed point and elliptic fixed point. Also compute the Hamiltonian $H(x,y)$ of the system and sketch the phase portrait (8 marks)

QUESTION THREE (15 MARKS)

a) Consider the system

$$x' = \lambda_1 x + f_1(x,y,z;\mu)$$
$$y' = \lambda_2 x + f_2(x,y,z;\mu)$$
$$z' = \lambda_3 x + f_3(x,y,z;\mu)$$

Where $(x,y,z,\mu) \in R^1 \times R^1 \times R^1 \times R^1$ and $\lambda_1, \lambda_2 < 0, \lambda_3 > 0$ are eigenvalues while $f_i \in C^2$ with the fixed point at $(x,y,z,\mu) = (0,0,0,0)$. Compute P_0, the flow of the linearized system about the fixed point, determine the time of flight from Π_0 to Π_1, and hence determine the map $P_1 : \Pi_0 \to \Pi_1$ (10 Marks)

b) Show that the system $x' = y,\ \ y' = x + x^2$ has a homoclinic loop S_0 at the saddle at the origin given by the motion on the curve $y^2 = x^2 + \frac{2}{3}x^3$ (5 Marks)

QUESTION FOUR (15 MARKS)

a) Construct the invariant Cantor set of a Horse shoe map defined as;

$\Lambda \equiv \bigcap_{j=-\infty}^{\infty} F^j(S)$ where S is the square domain, F is the horse shoe map which contracts S in the x direction, stretch S along the y direction and folds S back to itself, that is $F : S \to R^2$ and $S = \{x \in R : 0 \le x \le 1, 0 \le y \le 1\}$ (5 Marks)

b) Consider a two dimensional autonomous nonlinear system

$$x' = x - x^3$$
$$y' = -y$$

(i) Find all the fixed points or attracting sets and their nature (4 Marks)

(ii) Compute the Lyapunov exponent associated with orbits in the attracting set (6 Marks)

QUESTION FIVE (15 MARKS)

Consider the system

$$x' = 2y$$
$$y' = 12x - 3x^2$$

a) Find the Hamiltonian $H(x, y)$ of the system above (3 Marks)

b) Find all the fixed points and Sketch the phase portrait for $\alpha > 0$, of a negatively rotated vector field inside the graphic $H(x, y) = 0$ defined as; (5 Marks)

$$x' = X(x, y, \alpha) = y - \alpha H(x, y)(12x - 3x^2)$$
$$y' = Y(x, y, \alpha) = 12x - 3x^2 + 2\alpha H(x, y)y$$

c) With $\alpha = 0.1$, fixed, embed the vector field in (c) above in a one parameter family of rotated vector fields and determine for what values of μ will this system have;

(i) Limit cycle (3 Marks)

(ii) Experience a Hopf bifurcation (4 Marks)