(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR MAIN EXAMINATIONS

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN DISASTER PREPAREDNESS AND ENGINEERING MANAGEMENT

COURSE CODE: DPG 300
COURSE TITLE: NUMERICAL METHODS WITH MATLAB

TIME: 2 HOURS 12-2 PM DATE: 18TH DECEMBER 2023,

Instruction to the candidates:
Answer question ONE (COMPULSORY) and any other TWO questions
Time: 3 hours

This paper consists of 4 printed pages. Please turn over.

SECTION I: Answer ALL the questions in this section

QUESTION ONE - 30 MARKS (COMPULSORY)

(a) Construct a Lagrange interpolant of degree 3 for [5 mks]

x	-2	-1	1	3
y	16	1	1	81

(b) Explain Runge's phenomenon as used in interpolation. [2 mks]

(c) Derive and state the disadvantage of Euler's method for solving differential equations. [3 mks]

(d) Solve the differential equation $\frac{dy}{dx} = y + 3e^x;\ y(0) = 1,$ at $x = 0.2$ Using Taylor's series method. [5 mks]

(e) For the data $(-2, -6)$, $(-1, 0)$, $(1, 0)$, $(2, 6)$, construct a divided difference table. [5 mks]

(f) Solve the differential equations $\frac{dy}{dx} = 3x^4 - y,\ y(0) = 0.2$, at $x = 0.2$ using modified Euler's method correct to three decimal places with $h = 0.1$ [5 mks

(g) Find the area under the curve $f(x) = \sin(x)$ between 0 and $\frac{\pi}{2}$ using the trapezoidal rule with 5 points. [5 mks]

SECTION II: Answer any TWO questions from this section

QUESTION TWO - 20 MARKS

(a) Derive the Lagrange interpolating formula given two points (x_0, y_0) and (x_1, y_1). [5 mks]

(b) Construct a Newton interpolant of degree 3 for [5 mks]

x	0	0.5	1	1.5
$f(x)$	2	0	-1	3

(c) Write a MATLAB code for evaluating and plotting the function

$$f(x) = \frac{x^8 + 3x^4 + 3}{x^4 + 2}$$

. [5 mks]

(d) Approximate the function $f(x) = x^9 + 3x^2$ using a 5 point Newton interpolating polynomial in the interval $x = [-2, 2]$. [5 mks]

QUESTION THREE - 20 MARKS

(a) Use Gauss-Elimination with partial pivoting and scaling to solve the following system. **[5 mks]**

$$\begin{aligned} 3x + 2y + 100z &= 105 \\ -x + 3y + 100z &= 102 \\ x + 2y - z &= 2 \end{aligned}$$

(b) Find the solution of the system

$$\begin{aligned} 4x_1 + 6x_2 + 8x_3 &= 6 \\ 6x_1 + 10x_2 + 17x_3 &= 5 \\ 8x_1 + 17x_2 + 25x_3 &= 7 \end{aligned}$$

using Jacobi's method. **[8 mks]**

(c) Perform a QR decomposition of

$$A = \begin{bmatrix} 12 & -51 & 4 \\ 6 & 167 & -68 \\ -4 & 24 & -41 \end{bmatrix}$$

. **[7 mks]**

QUESTION FOUR - 20 MARKS

(a) Using Taylor series approximation, solve the differential equation $\frac{dy}{dx} = xy + \sin(y)$, $y(0) = 0.2$ at $x = 0.3$. **[5 mks]**

(b) Approximate $y(0.2)$ given the initial value problem

$$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 1$$

using

(i) Euler's method **[3 mks]**

(ii) Improved Euler's method with $h = 0.1$ correct to 3 decimal places. **[5 mks]**

(iii) Fourth order Runge Kutta method **[7 mks]**

QUESTION FIVE - 20 MARKS

(a) Interpolate the points (80, 25), (90, 30), (100, 42), (110, 50), (120, 68) using Newton interpolating polynomials. **[6 mks]**

(b) Approximate the integral

$$\int_0^{\pi} \sin(x)dx$$

using Newton Cotes formula with $n = 6$. **[7 mks]**

(c) Obtain a fourth-order accurate approximation of

$$\int_0^2 \cos(3x)dx$$

using Romberg integration. **[7 mks]**