(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

MAIN CAMPUS

UNIVERSITY EXAMINATIONS
2014/2015 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE
OF
MASTER OF SCIENCE IN WATER RESOURCES ENGINEERING

COURSE CODE: **CWE 800**

COURSE TITLE: **NUMERICAL METHODS**

DATE: **17TH DECEMBER 2014** **TIME:** **8.30AM – 11.30AM**

INSTRUCTIONS:

1. Answer any **FOUR** questions. All questions carry equal marks.
2. Examination duration is **3 Hours**

MMUST observes ZERO tolerance to examination cheating

QUESTION ONE

The following data for the density of nitrogen gas versus temperature come from a table that was measured with high precision.

T, K	200	250	300	350	400	450
Density, Kg/m^3	1.708	1.367	1.139	0.967	0.854	0.759

(a) Use first- through fifth-order polynomials to estimate the density at a temperature of 330 K.
(b) What is your best estimate?
(c) Employ this best estimate and inverse interpolation to determine the corresponding temperature.

QUESTION TWO

(a) derive Simpson Rule **(8 marks)**

(b) Water exerts pressure on the upstream face of a dam as shown in Fig. Q2. The pressure can be characterized by

$$p(z) = \rho g(D - z)$$

where p(z) = pressure in N/m^2 exerted at an elevation z metres above the reservoir bottom; ρ = density of water, which for this problem is assumed constant 10^3 kg/m^3; g = acceleration due to gravity (9.81 m/s^2); and D = elevation (in m) of the water surface above the reservoir bottom. Pressure increases linearly with depth, as depicted by Fig. Q2a. Omitting atmospheric pressure (because it works against both sides of the dam face and essentially cancels out), the total force f_t can be determined by multiplying pressure times the area of the dam face as shown in Fig. Q2b. Because both pressure and area vary with elevation, the total force is obtained by evaluating

$$f_t = \int_0^D \rho g w(z)(D - z)dz \qquad +++$$

where $w(z)$ = width of the dam face (m) at elevation z (Fig. 2a). The line of action can also be obtained by evaluating

$$D = \frac{\int_0^D \rho g w(z)(D - z)dz}{\int_0^D \rho g(D - z)dz}$$

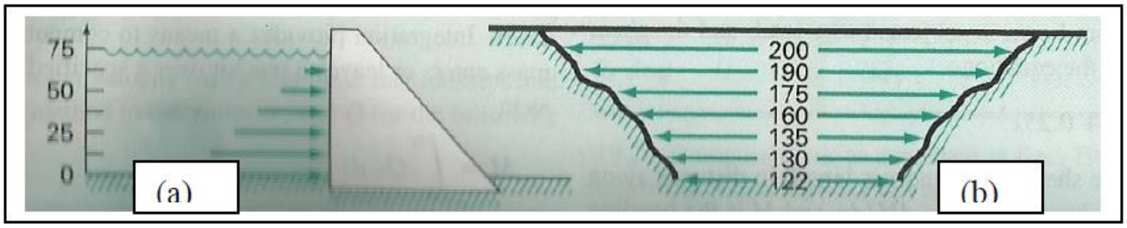

Figure Q2. Water exerting pressure on the upstream face of a dam: (a) side view showing force increasing linearly with depth; (b) front view shows width of dam in metres.

Use Simpson's rule to compute f_t and D (**17 marks)**

QUESTION THREE

Solve the following initial value problem over the interval from t = 0 to 2 where y(0) = 1.

$$\frac{dy}{dt} = yt^3 - 1.5y$$

(a) Analytically
(b) Using Euler's method with h = 0.5 and 0.25.
(c) Using the midpoint method with h = 0.5.
(d) Using the fourth-order RK method with h = 0.5.

QUESTION FOUR

Given the system of equations

$$2x_2 + 5x_3 = 1$$
$$2x_1 + x_2 + x_3 = 1$$
$$3x_1 + 7x_2 = 2$$

(a) Compute the determinant.
(b) Use Cramer's rule to solve for the *x*'s.
(c) Use Gauss elimination with partial pivoting to solve for the *x*'s. As part of the computation, calculate the determinant in order to verify the value computed in (a)
(d) Substitute your results back into the original equations to check your solution.

QUESTION FIVE

Use the Gauss-Seidel method without relaxation to solve the following system to a tolerance of ε_s = 5%. If necessary, rearrange the equations to achieve convergence.

$$2x_1 - 6x_2 - x_3 = -38$$
$$-3x_1 - x_2 + 7x_3 = -34$$
$$-8x_1 + x_2 - 2x_3 = -20$$