20

(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

(MAIN EXAMINATION)

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE
OF
BACHELOR OF SCIENCE

COURSE CODE: MAT 100

COURSE TITLE: MATHEMATICS FOR TECHNOLOGISTS

DATE: 15th Dec, 2023 TIME: 8 – 10 AM

Instructions to candidates:
Answer Question ONE (Compulsory) and ANY other TWO Questions

Time: 2 hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30MARKS) COMPULSORY

a) i) Prove that if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$ for the sets A,B and C. (3marks)
ii) Given $A = \{1,2,3,4,5\}$ and $B = \{4,6,1,2\}$ find $A \triangle B$. (2marks)

b) Let A be the statement: Jean went to the market and B be the statement: Jean travelled to Nairobi. Develop a truth table for the compound proportion $A \vee B$. (2marks)

c) i) Consider the functions $f: \mathbb{Z} \longrightarrow \mathbb{Z}$ and $g: \mathbb{Z} \longrightarrow \mathbb{Z}$ be defined by $f(n) = 2n + 3$ and $g(n) = 3n + 2$. Show that $f \circ g \neq g \circ f$. (3marks)
ii) Let $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ be two injective functions. Show that $g \circ f$ is also injective. (3marks)

d) An arithmetic progression whose first term is 3and nth is 171 has the sum of its first n terms equal to 3741. Find the value of n and the common difference. (3marks)

e) i) Given $p(x) = -2x^4 + 5x^3 + 4x - 7$. Describe the end behaviour of the polynomial. (3marks)
ii) Resolve into partial fractions $\frac{x^2+1}{x^2-3x+2}$. (4marks)

f) Find the area of a parallelogram with the adjacent sides $\boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{2j} - \boldsymbol{3k}$ (3marks)

g) i) Find the determinant of the matrix $A = \begin{bmatrix} 1 & -2 & 0 \\ -3 & 5 & 1 \\ 4 & -3 & 2 \end{bmatrix}$. (2marks)
ii) Given matrices A and B, evaluate $(A + B)^2$ and $(A + B)(A - B)$. (2marks)

QUESTION TWO (20MARKS)

a) Find the magnitude and the direction cosines of vectors below. (4marks)
$3\boldsymbol{i} + 7\boldsymbol{j} - 4\boldsymbol{k}$, $\boldsymbol{i} - 5\boldsymbol{j} - 8\boldsymbol{k}$ and $6\boldsymbol{i} - 2\boldsymbol{j} + 12\boldsymbol{k}$.

b) Show that if $f: X \longrightarrow Y$ is one-to-one correspondence then $f^{-1}: Y \longrightarrow X$ is also one-to-one correspondence (6marks)

c) Let $p(x) = x^3 - x^2 - 3x$
i) Find the zeros of $p(x)$. (2marks)
ii) Sketch a graph of $p(x)$ (5marks)

d) Given sets A,B and C such that $(B \cap C^c) \cap A^c$. Use Venn diagram to show this relation. (3marks)

QUESTION THREE (20MARKS)

a) Given that $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 1 & 0 & 3 \end{pmatrix}$. Find A^{-1} using the cofactors of A. (10marks)

b) Solve by Cramer's Rule (10marks)

$$5x - 8y + z = 11$$
$$6x - 8y - z = 15$$
$$3x + 2y - 6z = 7$$

QUESTION FOUR (20 MARKS)

a) Evaluate the values of the following definite and indefinite integrals.

i) $\int_0^2 \frac{3x}{\sqrt{2x^2+1}} dx$ (5marks)

ii) $\int_1^3 5x\sqrt{(2x^2 + 7)}\ dx$ (5marks)

iii) $\int \frac{(1+\theta)^2}{\sqrt{\theta}} d\theta$ (4marks)

b) Resolve into partial fractions $\frac{-2x+4}{(x^2+1)(x-1)^2}$ (6marks)

QUESTION FIVE (20 MARKS)

a) Given simple propositions A,B and C. Show the truth table for the compound proposition $[(A \Rightarrow B) \wedge (B \Rightarrow C)] \Rightarrow (A \Rightarrow C)$. (7marks)

b) Find the sum of n-terms of the series. (6marks)

$$5 - x + \frac{x^2}{5} - \frac{x^3}{5^2} + \cdots + \frac{(-1)^{r-1}x^{r-1}}{5^{r-2}} + \cdots n$$

c) An arithmetic sequence has first term *a*, and a common difference *d*. it is given that the sum of the first four terms is more than the sum of the next four terms by 8. Also, the first term, third term and sixth term of the sequence are three consecutive terms of the GP. Find exact value of *a* and *d*. (7marks)