70

(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

MAIN CAMPUS

UNIVERSITY REGULAR EXAMINATIONS
2023/2024 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE
OF
BACHELOR OF SCIENCE IN CIVIL AND STRUCTURAL ENGINEERING

COURSE CODE: CSE 311

COURSE TITLE: FINITE ELEMENT METHOD

DATE: 11TH DECEMBER 2023 TIME: 12 P.M – 2 P.M

INSTRUCTIONS:

1. This paper contains **FIVE** questions
2. Answer **QUESTION ONE** and any other **TWO** Questions
3. Marks for each question are indicated in the parenthesis.
4. Examination duration is **2 Hours**

MMUST observes ZERO tolerance to examination cheating

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (*30 MARKS*): COMPULSORY

a) With illustration, explain the fundamental concepts of finite element method. (**4 Marks**)

b) State any FIVE advantages of the finite element method. (**5 Marks**)

c) Given a two-node bar element, derive the shape functions at each node using polynomial functions. (**6 Marks**)

d) Using the shape functions derived in (c), define the term 'shape function' and hence illustrate THREE properties of a shape function. (**7 Marks**)

e) From the shapes functions derived in (c) above, show that the stiffness matrix of a two-node bar element is given by:

$$k^e = \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

(**8 Marks**)

QUESTION TWO (*20 Marks*)

For the beam and loading shown in Figure Q2, calculate the rotations at B and C. Take $E = 210GPa$ and $I = 6 \times 10^6 \text{mm}^4$. Use the element stiffness matrix given by:

$$[\text{K}] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

(**20 Marks**)

QUESTION THREE (*20 Marks*)

Consider a three-member truss shown in Figure Q3. All members of the truss have identical areas of cross-section, A, and modulus of elasticity, E. Determine the horizontal and vertical displacement at the joint A using finite element method. (**20 Marks**)

Take:

$$[K^e] = \frac{E_e A_e}{l_e}\begin{bmatrix} \cos^2\theta_e & \frac{1}{2}\sin 2\theta_e & -\cos^2\theta_e & -\frac{1}{2}\sin 2\theta_e \\ \frac{1}{2}\sin 2\theta_e & \sin^2\theta_e & -\frac{1}{2}\sin 2\theta_e & -\sin^2\theta_e \\ -\cos^2\theta_e & -\frac{1}{2}\sin 2\theta_e & \cos^2\theta_e & \frac{1}{2}\sin 2\theta_e \\ -\frac{1}{2}\sin 2\theta_e & -\sin^2\theta_e & \frac{1}{2}\sin 2\theta_e & \sin^2\theta_e \end{bmatrix}$$

where the symbols are standard symbols used in finite element analysis

QUESTION FOUR (*20 Marks*)

A triangular finite element in a domain under 2-dimensional analysis is as shown in Figure Q4. The nodal coordinates give the geometry of the element in meters. Under applied loads, the nodal solution gives displacements, in millimeters, at each of the nodes as follows;

$$u_1 = \begin{Bmatrix} 3.5 \\ -7.0 \end{Bmatrix}; \qquad u_2 = \begin{Bmatrix} 5.0 \\ 15.0 \end{Bmatrix}; \qquad u_3 = \begin{Bmatrix} -4.5 \\ -7.5 \end{Bmatrix}$$

Where u_i is the vector of horizontal and vertical displacements at the respective nodes.

a) Calculate the shape functions for the element. **(15 Marks)**

b) Calculate the displacements in point P. **(5 Marks)**

Take: $N_i = \frac{1}{2A}[a_i + b_i x + c_i y]; \quad a_i = x_j y_k - x_k y_j; \quad b_i = y_j - y_k; \; c_i = x_k - x_j$

$$2A = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}$$

QUESTION FIVE (*20 Marks*)

(a) State and explain the major steps in solving structural problems using the finite element method.
(11 Marks)

(b) A complex shape is to be analysed using one triangular element and one rectangular element as shown in Figure Q5.

i) State the two principles used in the assembly of the finite element equations. **(2 Marks)**

ii) Provide the assembled stiffness matrix for the two-element mesh shown in Figure Q5. **(7 Marks)**

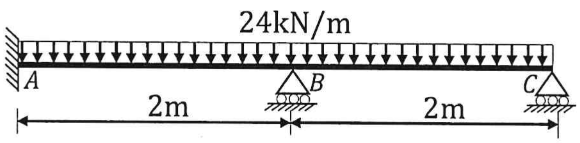

Figure Q2

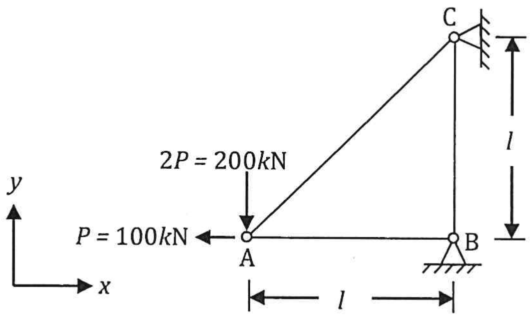

Figure Q3

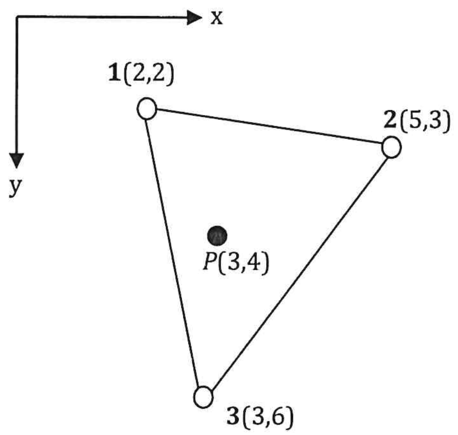

Figure Q4

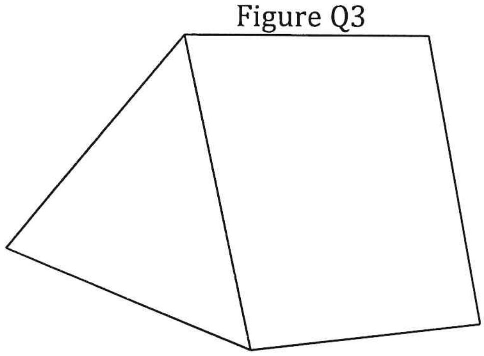

Figure Q5

======*END OF PAPER*=====