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(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

MAIN CAMPUS

UNIVERSITY EXAMINATIONS 2023/2024 ACADEMIC YEAR

FIFTH YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL AND COMMUNICATION ENGINEERING

COURSE CODE: ECE 513

COURSE TITLE: NON-LINEAR AND MULTIVARIABLE CONTROL

DATE: THURSDAY 14/12/2023 **TIME:** 12: 00 PM – 2:00 PM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS.
QUESTION ONE CARRIES 30 MARKS AND ALL OTHERS 20 MARKS EACH.

TIME: 2 Hours

MMUST observes ZERO tolerance to examination cheating

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Consider the function given below.

$$Y[n] = \sum_{k=0}^{n^2} \cos\left(\frac{\pi}{4}k\right) x[k]$$

b) Test whether the system is linear or nonlinear

[4 Marks]

c) Briefly highlight on the at least three essentially nonlinear phenomena

[6 Marks]

d) List at least six methods of analyzing nonlinear systems

[3 Marks]

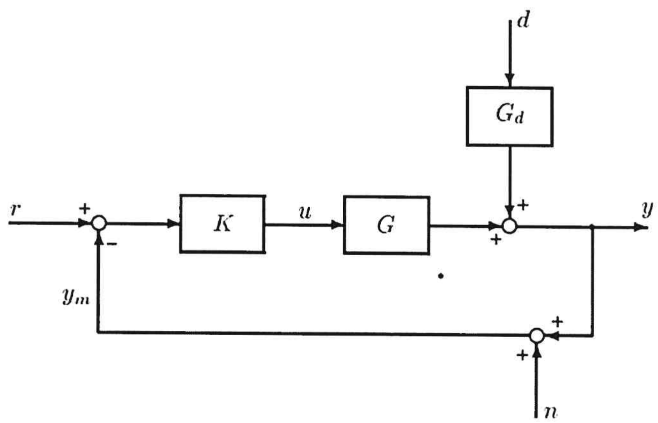
e) What is an optimal control system and list the necessary information required to formulate the problem of optimization of a control system.

[3 Marks]

f) Differentiate the following techniques as architectures of nonlinear control
   i. Feedback Linearization and Backstepping
   ii. Bounding Control and Sliding Mode Control
   iii. Gain Scheduling and Adaptive Bounding Control

[9 Marks]

g) Consider one degree of freedom feedback control system below, Compute the following



   i. The closed loop response
   ii. The control error
   iii. The corresponding plant input signal

[5 Marks]

QUESTION TWO (20 MARKS)

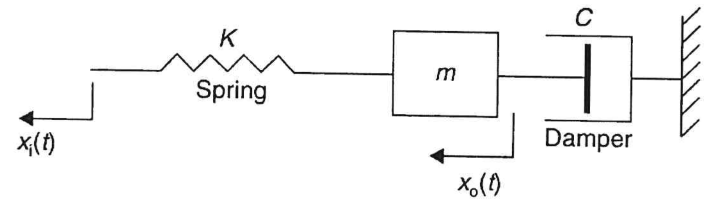
a) Consider a nonlinear system given by equation

$$3x + 4y = z$$

   i. Linearize the nonlinear equation in the region $6 \leq x \geq 8, 9 \leq y \geq 11$.
   ii. Find the error if the linearized equation is used to calculate the value of z when $x = 6, y = 9$

[8 Marks]

b) Consider the spring-mass- damper system as shown in the Figure below. If $x_i(t)$ and $x_0(t)$ represents the input and output displacements; while K is the spring constant, C is the damping coefficient and m is the mass.



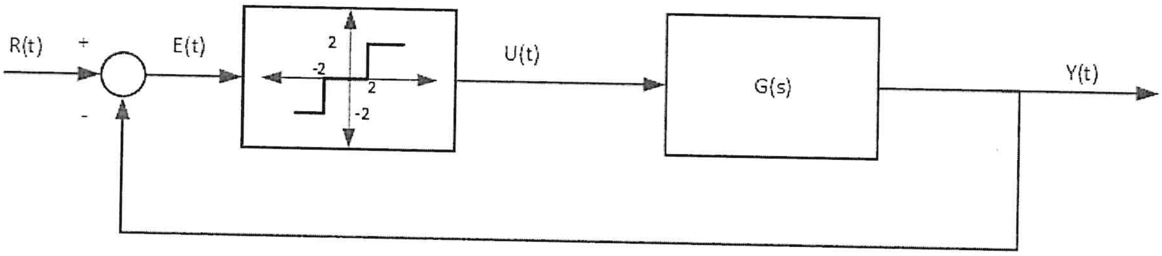
i. Obtain the mathematical model of this system.

ii. Determine if the system is stable or unstable in the sense of Lyapunov

[12 Marks]

QUESTION THREE (20 MARKS)

a) Consider a nonlinear element in a closed loop control system below. If $G(s) = \frac{1}{s(s+1)(s+5)}$



i. Derive the describing function of the nonlinear element

ii. Obtain the state equations

iii. Obtain the equations for state trajectories phase plane

iv. Does limit cycle exist

v. if so determine if the limit cycle is a sustained oscillation

vi. If the limit cycle exists, find the amplitude and frequency of the limit cycle.

[18 Marks]

b) List two assumptions made in the describing function analysis

[2 Marks]

QUESTION FOUR (20 MARKS)

a) Consider the nonlinear differential equation

$$\ddot{y} - \left(0.2 - \frac{20}{6}\dot{y}^2\right)\dot{y} + y + y^2 = 0$$

i. Find all the singularities of the system

ii. Classify all singularities

iii. Sketch the phase portrait in the neighborhood of the equilibrium points

[12 Marks]

b) Consider the nonlinear differential equation below

$$\ddot{x} + \dot{x} + 4\cos(x)\dot{x} + 3sin(x) = 0$$

i. Determine the linearized system of equation in the phase plane

ii. Determine the location of the eigenvalues for the linearized system of equations

[8 Marks]

QUESTION FIVE (20 MARKS)

a) Given a scalar function $V(x) = 10x_1^2 + 4x_2^2 + x_3^2 + 2x_1x_2 - 2x_2x_3 - 4x_3x_1$

i. Represent the scalar function $V(x)$ in quadratic form

ii. Determine the definiteness of (ii) based on Sylvester theorem

[6 Marks]

b) Consider a system described by

$$\begin{bmatrix} \dot{x}_1 = -x_1 + x_2(x_1^2 + x_2^2) \\ \dot{x}_2 = x_2 + x_1(x_1^2 + x_2^2) \end{bmatrix}$$

Check for stability using Lyapunov's 2nd method apply both 1st and 2nd theorem.

[6 Marks]

c) Consider a system defined by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

If the performance index is given by

$$J = \frac{1}{2} \sum_{k=0}^{\infty} [\mathbf{x}^*(k)\boldsymbol{Q}\mathbf{x}(k) + u(k)Ru(k)]$$

Where $Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $R = 1$.

a) Determine the optimal control law to minimize the performance index

b) Compute the minimum value of J.

[8 Marks]