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(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS 2023/2024 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ECONOMICS

COURSE CODE: ECO 903

COURSE TITLE: MATHEMATICAL ECONOMICS

DATE: FRIDAY 26TH April 2024 **TIME:** 9:00 -12:00

INSTRUCTIONS TO CANDIDATES

ATTEMPT QUESTION **ONE** AND ANY OTHER **THREE**

TIME: 3 Hours

MMUST observes ZERO tolerance to examination cheating

QUESTION ONE (COMPULSORY)

a) Phase diagrams are very useful in conducting qualitative analysis given differential equations, explain what a phase diagram is and examine its usefulness. (3 marks)

b) The Leontief inverse of an economy with three sectors is given by

$$[I-A]^{-1}=\begin{bmatrix}1.7 & 0.25 & 0.33\\ 0.72 & 1.47 & 0.52\\ 1.18 & 0.59 & 1.76\end{bmatrix}$$

i. The government has planned a final demand of 800, with sectoral components being $D_1=300$, $D_2=300$ and $D_3=200$. How much total sectoral output will be required to realize this level of final demand? (5 marks)

ii. How much (a) sectoral primary inputs and (b) total primary inputs will be necessary for the realization of this final demand using the inner product concept for vectors of sectoral and total primary inputs, given the following primary input coefficients; $[a_{v1} \quad a_{v2} \quad a_{v3}]=[0.07 \quad 0.03 \quad 0.05]$. (7 marks)

c) Find the characteristic equations and the corresponding characteristic roots for the following;

i. $\frac{d^2y}{dt}+3\frac{dy}{dt}+5y=0$ (5 marks)

ii. $\ddot{x}-6\dot{x}+3\frac{1}{4}x=0$ (5 marks)

d) Consider a closed economy model where C- Consumption, Y-Income, I – Investment and G-government expenditure such that;

$Y=C+I+G$

$C=100=0.6Y$

$I=50+0.2Y$

$G=500$

Required;

i. Write the model in the matrix format $\underset{3\times3}{A}\ \underset{3\times1}{X}=\underset{3\times1}{B}$ (4 marks)

ii. Compute the second order principal minors for A. (3 marks)

iii. Determine the trace and rank of A. (3 marks)

iv. Compute equilibrium income, consumption and investment. (5 marks)

QUESTION TWO

a) Using an appropriate diagram to analyze the dynamic stability of the following time paths;

i. $X(t)=5-2e^{-\frac{1}{2}t}$ (5 marks)

ii. $u(t)=5t+7$ (5 marks)

b) Two risk averse consumers A and B have the following utility functions. Find out which of these individuals is relatively more risk averse than the other. (10 marks)

i. $U(x)=\frac{4}{7}x^{\frac{7}{8}}$

ii. $U(x)=\frac{1}{5}x^{\frac{1}{8}}$

QUESTION THREE

a) Find the time path of P for the following market model with expectation. Extract the differential equation in P. (3 marks)

$\overline{Q}_d(t)=\overline{Q}_s(t)=\overline{Q}(t)$

$\overline{Q}_d(t)=10-P-\dot{P}-0.5\ddot{P}$

$\overline{Q}_s(t)=-3+2P$

b) Two producers have the following production functions, state the homogeneity of the functions and demonstrate Euler's Theorem. (10 marks)

i. $Q=AK^aL^b$

ii. $Q=AK^2L$

c) An agriculturalist has a 125-acre farm. He produces radish, maize, and potato. Whatever he produces is fully sold in the market. He gets Ksh.5 for tea per kg, Ksh.4 for maize per kg and Ksh.5 for potato per kg. The average per acre yield is 1,500 kg of radish, 1,800 kg of maize and 1,200 kg per potato. A sum of Ksh.12.50 has to be used for manure to produce 100 kg each of tea and maize and 80 kg of potato. The labour required to produce the crop for each acre is 6 man-days each for tea and potato and 5-man-days each for maize. A total of 500 days of labour at a rate Ksh.40 per man-day are available. Formulate this as a linear programming problem to maximize the agriculturalist's profit without solving. (7 marks)

QUESTION FOUR

a) Determine whether the following function is continuous and differentiable at $x=2$; $f(x)=|x-2|$ (7 marks)

b) Determine consumer surplus given that equilibrium price (P = 3) for the following demand and supply functions. (3 marks)

$Q_d = 10 - 2P \quad Q_s = -2 + 2P$

c) The national income model can be expressed as a difference equation. Considering the following model given as;

$Y_t = C_t + I_t$

$C_t = 100 = 0.9Y_{t-1}$

$I_t = 50$

$Y_0 = 2000$

Required;

i. Solve for, the time path of income. (4 marks)

ii. Determine whether the economy is stable or unstable. (3 marks)

iii. Compute the intertemporal equilibrium value of income. (3 marks)

QUESTION FIVE

Consider the following linear autonomous first order equation $\dot{Y}(t) = -\frac{1}{2}Y(t) + 3$

i. Calculate the steady state value of Y. (4 marks)

ii. Determine whether or not the steady-state equilibrium value is dynamically stable. (4 marks)

iii. Use the phase diagram to analyze the stability of the system. (5 marks)

iv. Find the general solution of the equation and graphically present the time path of $Y(t)$. (7 marks)